EXPLAINING GRAVITY

AND

HUBBLE COSMOLOGY

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PREFACE

I came up with an idea on how to explain gravity. To compare my idea with other theory, I enrolled in a class at the University of Oregon to learn Einstein's theory of special relativity. I challenged it to understand it, and I determined it as internally consistent. Shortly thereafter, however, I did not find General Relativity to be consistent with Special Relativity.

By Special Relativity, light speed is a limiting condition for matter not to reach. To the contrary, General Relativity now relates to black holes and a singularity, even though Stephen Hawking redefined the black hole for it to emit Hawking radiation, as to be consistent with thermodynamics and quantum physics. The inconsistency remains because special relativity is a unification of mechanics and electrodynamics, and because Einstein did not include general relativity in this unification.

Einstein assumed the universe is finite and static, but his equations did not comply as static according to Alexander Friedmann. Moreover, with the discovery of the redshift in more distant starlight, a cosmological principle was proposed for an expanding universe theory. An alternate interpretation of the redshift was also proposed as Tired Light theory, but it was discarded in favor of Big Bang in view of a Tolman Brightness Test. Even though astronomical data later failed to support the test in favor of Big Bang, as for it to now claim expansion of the universe has increased by means of "dark-energy", Tired Light is still dismissed. It is now argued that it does not explain visibility of the distant stars as to why interaction of light with the space medium does not alter the path of light rays in various directions.

Explanation is here given. Moreover, tired light along with the Hubble Constant provides long-range effect of gravity whereby a Hubble Constant change in speed per light speed for a distance equal to the diameter of the hydrogen atom equals the ratio of gravitational to electrostatic force of the hydrogen atom, and there is a recycling process for maintaining matter. This explanation is simple, consistent and complete according to a historical development of theory apart from Big Bang.

The history of physics is a means of understanding its development as a step by step process. However, the history generally contains the language of mathematics that is too foreign for some of us to understand. There is a multitude of languages; such as, different systems and different units of measurement. There are thus newtons, farads, amperes, coulombs, ergs and so forth. There are even systems of dimensionless units, such as plank units.

Physicists have claimed a supercomputer is needed to solve Einstein's general field equations, but the computer programming is based on a binary code, a twonumber system, whereby it is possible to relate the general field equations of general relativity as a step by step process, but the steps in this case are too numerous, as for them to fill volumes of books. The math allows us to accurately describe specific aspects of nature, and math complexity also simplifies the tasks of experts in the field more knowledgeable of its usage, but higher math is not needed for a fundamental understanding of theory.

A complete understanding of all this seems out of reach for anyone of us lacking in higher education, but Einstein suggested there is a simpler step for understanding relativity theory in view of the Pythagorean Theorem: ($C^2 = A^2 + B^2$). An effort has thus been made to simplify all the mathematics in this book in order that it is not any more difficult to understand than is the Pythagorean Theorem, as of simple algebra and geometry.

Algebra is numerical math simplified. It should be taught early in school along with arithmetic. Its simplicity is about such symbols as letters of the alphabet substituted for numbers. For instance, in place of adding numbers, say 156 and 44, in the manner 156 + 44 = 200, chosen letters are substituted in the manner A + B = C. Instead of multiplying such numbers as 3 and 4 in the manner $3 \ge 4 = 12$, symbols are presented in the manner AB = C. And $3 \ge 3 \ge 3^3 = 27$ can be represented as say C³.

There is also a convenience of symbolic algebra for solving unknowns. For instance, if 5A + 4B = 6C and 5A - 4B = 2C, then the simple steps of addition, subtraction, multiplication, division, and substitution are a means of obtaining numerical results. Adding the two equations obtains

$$5A + 4B = 6C$$
$$5A - 4B = 2C$$
$$10A + 0 = 8C$$
$$10A = 8C$$

Dividing 10 by 8 and reversing order obtains

$$C = 1.25A$$

Substituting obtained values in one of the equations obtains

$$5A + 4B = 6C = 7.5A$$

 $4B = 7.5A - 5A = 2.5A$
 $B = 0.625A$

Thus, all numerical values are obtainable by obtaining a numerical value of either A, B, or C.

Algebra is particularly helpful for solving such geometrical problems as proving the Pythagorean Theorem whereby the length of the hypotenuse of a right triangle equals the square root of each perpendicular length squared and then added in the manner $C^2 = A^2 + B^2$. To illustrate, consider a larger square $(A + B)^2$ whereby its side lengths A plus B are four extensions of the lengths of perpendicular sides of four equal right triangles that, along with a square of the length C of the hypotenuse, are within the larger square. The geometry is illustrated in the manner



Note: Simple steps of addition, subtraction, multiplication and substitution are here implied, as left for the reader to apply.

INTRODUCTION

Rene Descartes philosophized, "I think, therefore I am. If I am deceived of my existence, then I must at least be the Deceiver." He therefore confirmed his existence by way of his own awareness of himself.

Am I alone?

Resistance to thought and action is testimony to an extended existence of an objective world in which we live. I am not alone. We therefore exist.

I witness existence, but I do not seem to know how either it or a state of consciousness is possible. I merely assume we are parts of creation, as by either a Supreme Being or from what merely exist.

Can there be existence without our awareness of it? No matter. Never mind. The mind-matter duality is of no concern, as this book is about the physical world inasmuch as material existence is comprised of a substance of some sort that varies in shape, size, density and so forth. In this regard, philosophers have attempted to explain everything existing in the world as part of a primary substance that became referred to as æther.

Suppose all physical reality is indeed comprised of æther. What, then, are its properties? How does it, for instance, separate from and recombine with itself in creating effects of reality? A substance being primary implies it has no internal mechanism to bond with other primary substance except for it interacting with its other parts. If such interaction is relative motion, then the action must somehow be elastic to maintain itself, as action between itself would otherwise result in a loss of relative motion.

Another likely condition for primary substance is that it needs to be of infinite content, even if only to partially fill infinite space instead of all of it, for it to change direction by means of elastic collision instead of spreading apart without a means to reverse direction to again interact with itself. The question then comes to mind as to whether space is a plenum of æther or a partial vacuum and æther. Descartes, along with other philosophers, assumed space consists of a plenum. How, then, do various densities of matter exist if æther is everywhere identical in composition?

A plenum does not even have wiggle room for wave action to occur in the manner sound waves are compressed states of air molecules. The æther as a plenum is thus contrary to the air medium for sound.

It is no wonder that early Greeks deep in thought regarded our world as illusionary. As for the plenum, they considered only circular motion in it is possible. The sensation of free motion in various directions is allowed by the complexity of circular paths for our witness of effects.

Descartes also assumed all motion is circular as a complex system of vortices of diverse sizes and rotational speeds, and he further assumed total motion is conserved, as allowed by an exchange of motion between various actions of æther. As for the complexity of motion, endless possibilities exist by means of vortices moving inside other vortices for various relationships. A number of like vortices in one region of space can thus be relatively more or less dense than other vortices in another region of space. Our perception of the world thus comes about as an exchange of motion between vortices even though primary substance remains everywhere in space the same.

The source of all creation is thus already created out of what already exists ad infinitum. However, whatever existence we are aware of could be only part of many realities. Other parts of existence could, in effect, be invisible to us.

The æther is not presently accepted as scientific, as for being invisible if it does exist, but modern quantum mechanics is somewhat similar to it. In place of the æther, there is vacuum space that is not empty, as consisting of a virtual field of virtual particles detectable according to probability, as to be mathematically predictable and thus confirmable according to observation. However, indirect perception of the atomic world is different from that of the natural world of direct observation. The main difference is with regard to change. Interaction within the atomic world is of discrete units of energy, as change in effect, whereas change of particle interaction, either by relative motion or gravity, appears to be of a continuous nature.

To the contrary, this book provides explanation of relative motion and gravity effects consistent with quantum effects. The explanations are a step by step process in view of the historical development of theory for there to be no contradiction of quantum mechanics and relativity theory.

In the following chapter, Æther and Laws of Motion, the early history of Aristotelian physics led a step by step process to the discovery of laws of nature. Light, according to the early physics, provided earthly substance with the energy needed to maintain motion. In response, impetus theory evolved whereby matter maintains an innate ability to sustain motion until impeded by some obstacle as other matter. From two different viewpoints, space was either a plenum or a partial vacuum. Copernicus and Kepler promoted the latter with the heliocentric theory of planetary motion around the sun, and Galileo further explained natural laws of motion in view of a vacuum state, but the plenum was defended by such other philosophers as Gassendi and Descartes. They regarded the existence of an internal mechanism as essential for giving rise to the properties of the observable world. Although the laws of nature have by themselves provided much understanding of nature, there is still the internal mechanism of reality as inclusive to its understanding. A ratio of mass to volume space of virtual particles, for instance, relates to chemistry and biology, and the ratio of mass to volume space of some virtual particles lead to other ratios, such as that of the proton and electron.

In the next chapter, Newtonian Mechanics, it is explained according to Kepler's laws of planetary motion and Galilean relative motion, but not as a complete theory by itself. Newton, himself, was dissatisfied that he was only able to explain gravity according to an action at a distance principle. He also assumed ideal conditions of absolute space and absolute time that were later modified by Einstein. His mechanics also lacked a definition of energy that was later theorized according to the laws of thermodynamics. Nonetheless, even though Newtonian Mechanics was to be modified by Einstein for it to comply with the relativity of spacetime, it is still an integral part of theory as needed for a more complete understanding of theory.

In the next chapter, Kinematics Atoms and Electrodynamics, they are explained in the historical context of Boyle's Law discovered with the aid of several inventions that give rise to explaining heat and temperature in view of the kinematics of relative motion, which eventually led to atomic theory and the laws of thermodynamics. The second law is known as entropy. It is significant in describing a dormant and potential energy source whereby it was later used to verify the Fourth Power Law of Black Body radiation, and to become part of the development of quantum theory.

The next chapter, From Wave Theory to Relativity, is about a wave theory of light in contradistinction to the partial vacuum of space. The theory developed around 1900 to explain such properties as diffraction, but its requirement of light waves being transverse waves required explanation itself as to how transverse waves can exist in a three-dimensional medium in contrast to surface waves or waves along a rope. A solution was provided by Maxwell's electromagnetic theory, as light waves being a continuation of magnetic fields induced by electric currents and vice versa in free space. The speed of light was included in the theory as a constant. Constant speed through the æther was assumed, but the state of the æther remained questionable as whether it was dragged along with mass or was an absolute state not influenced by the presence of mass. Experiment indicated that light speed measures the same regardless of the relative motion of the system by which it is measured. An explanation was given by Lorentz whereby contraction of material length in the direction of relative motion and clocks being slowed by their motion in the æther contributed to the measure of light speed as constant. Lorentz did not conclude a variable æther state is undeterminable. He left that conclusion to Einstein, who further proclaimed æther is thereby of no use for the formulation of theory. It thereby became non-existent, as for physics to describe and explain only the natural world, but Einstein also suggested the æther can provide a deeper understanding of the nature of the universe. A loss of light energy to the æther by the transverse vibrations of light, for instance, can explain how light waves lose energy and maintain visibility of distant stars and galaxies. The lack of such explanation has been a criticism of tired light theory that is now defended.

In the next chapter, Simple Spacetime Relativity, the relativity of space, time and motion is shown to be an internally consistent theory according to the postulates of covariance and constant light speed. Consistency of theory includes the principle of simultaneity, the addition of velocities theorem, the clock paradox, the Doppler Effect, and constant speed change.

In the next chapter, Mass-Energy Dynamics, special relativity is shown consistent with such conservation laws of momentum and energy. The only questionable issue is about perception of the universe at large regarding a change in velocity of the observer. Although conservation laws apply to any systems interacting with each other, they do not apply with regard to how non-interacting systems are perceived anew from a changed state of the observer. For slower clocks and shorter rods, the universe at large could appear relatively larger, more massive and energetic unless perception of the observable part of the universe is relative as well.

The next chapter, The Relativity of Gravity, is more complex. Einstein attempted to generalize the principle of covariance to include gravity, but it is complicated by the inhomogeneous nature of gravity, such that gravity is described according to spacetime curvature due to the presence of mass. An energy-momentum tensor is used to describe spacetime curvature, but there are nonetheless conditions of relativistic effects of general relativity that are analogous to those of special relativity. Relativistic effects relating to the inhomogeneous nature of gravity compare with those of relative motion as acceleration, whereas homogeneous conditions of gravity, such as distance and time increments, are analogous to inertial motion.

A controversial issue of general relativity is regarding the universe as finite and expanding. Einstein considered a finite and static universe, but Friedmann pointed out to Einstein that a static universe was not stable according to the general field equations, even with Einstein's insertion of a Cosmological Constant. With the discovery of the redshift in more distant starlight, which indicated an expanding universe, a metric was proposed by Friedmann and others that contained a Cosmological Principle of isotropy and homogeneity, but the principle is here shown not to be consistent with relativity theory. The principle contains the condition of a singularity whereby an infinite mass-energy-density once existed within a no-volume space from which the universe is assumed to be expanding. However, by analogy of general relativity effects with special relativity effects, the singularity is of infinite mass-energy instead of only infinite mass-energy-density, but the expanding universe is only assumed to be of finite mass-energy. Moreover, the expanding universe theory is further complicated with the assumptions that Dark Energy and Dark Matter exist to explain why data of astronomical observation differs from the established laws of physics.

Another issue of general relativity is about infinities. They are resolved by quantum physics by means of renormalization. It has not been explained how general relativity can renormalize, but how light speed can be a limiting condition of gravity in analogy to that of special relativity is explainable. Special relativity is recognized as a unification of the Mechanics of relative motion along with electromagnetism, which includes light speed. After formulating general relativity, Einstein endeavored the remaining years of his life to formulate a unified field theory including gravity as well. He had been nearly there, needing only to pursue analogies of relative motion and gravity, but no inclusion of an addition of gravitational potentials theorem as analogous to the addition of velocities theorem of special relativity, as for resolving the issue of infinities, occurred.

In the next chapter, Quantum Origins, a Planck Constant is explained in the historical context of theorizing the natures of heat and light. Included is the Stefan-Boltzmann fourth power law that further relates to the nature of the atom. The laws of thermodynamic apply, particularly that of entropy regarding how temperature varies with force, as with further regard to Boyle's Law.

In the next chapter, Quantum Physics, it is explained how the Planck Constant formulates into theory. Included is Bohr's theory of the atom, the photoelectric effect explained by Einstein, the Hamilton Wave Mechanics, a wave interpretation of light and mass by de Broglie, the Schrodinger Wave Mechanics inclusive of the Planck Constant, a probability interpretation of the Schrodinger Wave Mechanics according to the Heisenberg uncertainty principle, and an antimatter complement to matter according to a symmetry proposed by Dirac. Significant is Dirac's unification of Quantum Wave Mechanic with special relativity and a fractional number one half that is used to explain angular momentum of atomic particles as spin, as it is also shown that the fraction one-half is unique for including general relativity in the unification as well. General relativity and Quantum Electrodynamics are both shown to comply with the Heisenberg uncertainty principle. They thereby equate.

The next chapter, The Relativity of Hubble Cosmology, addresses the inconsistency of the expanding universe, as Big Bang, and the Cosmological Principle. Explained more consistent instead is Tired Light. Along with a counter explanation of it as consistent with effects of the Tolman Brightness Test, it is mathematically more consistent in equating the Hubble Constant with average mass density of the universe and the ratio of gravity to electrostatic force. It is explained according to the probability condition of quantum physics. Indicative of the Hubble Constant is a minute change in energy per distance consistent with long-range gravitational effect whereby gravity itself is explained in view of the Hubble Constant. The last chapter, Gravity Cause Explained, indeed explains the cause of gravity along with electrostatic attraction and so forth. Another objection to Tired Light theory is addressed. Besides a Tolman Brightness Test, how stars of long distance are observed with no distorted Tired Light effects due to light interacting with its space medium has also been a reason to discard Tired Light theory. Explanation of the visibility of the stars pertains to a wave-particle paradox and a challenge to a Copenhagen Doctrine.

The Copenhagen Doctrine is a strict interpretation of quantum physics referring to the condition of probability. It was advocated by Heisenberg, Born, Bohr and other leading physics of the time. According to it, the only valid explanation of theory is that verifiable by observation in contrast to casual explanation. However, some verification is indirect, as by virtual particles necessary for the more accurate prediction of mathematical theory.

Such physicists as Einstein, de Broglie, Bohm and Vigier opposed the Copenhagen Doctrine in favor of a more inclusive casual explanation of the underlying causes of gravity and so forth. Explanation of the particle-wave-paradox was attempted by them. Vigier attempted to explain the visibility of the distant stars. Explanation is here given of the latter and of the cause of gravity as well. It includes explanation of a right-hand rule that itself explains why two wires with electric currents flowing in the same direction attract each other. Explaining the right-hand rule is according to zero-point-energy, as a modified version of quantum theory proposed by Planck. Zero-point-energy relates to virtual energy of vacuum space. Explanation of its effect is according to classical laws of motion, as consistent with the intent of Plank's modified version of quantum theory.

Gravity and electric charge are explained consistent with observing the natural world of continuous change that a quantum interpretation need not deny. According to one possible quantum interpretation, for instance, mass emits the same light it absorbs in restoring its original energy state. This is a historical principle that led to quantum theory itself. However, the original states are not necessarily restored. With the atomic structure, an interaction occurs with virtual particles for a more complex state of existence that can be explained in manner of its complexity in converting to various forms and emitting radiation that is eventually recycled back in manner of maintaining the quantum forms of mass and energy according to relative motion, gravity and electrostatics. Included in the explanation is the Hubble Constant, as in relation to the perception of the observable universe at large. However, the latter is more in favor of tired light rather than Big Bang theory.

In Quantum Origins, there is a blackbody condition whereby systems emit the same radiation they absorb. In Quantum Physics, there is also the photoelectric effect whereby a metal emits more electrons according to light frequency instead of its intensity. It was interpreted by Einstein as a particle effect. However, light frequency is both a wave and particle property. When light collides with a metal, it is semi-elastic in the sense the light itself is not reflected but is replaced by electrons instead. Inelasticity generally occurs when light is not too quick to neither reflect nor for electron replacement. It is then absorbed as heat energy that passes onto other systems. Similarly, the inelastic nature of gravity is such that mass-energy is converted to light-energy. The light emitted is, in turn, gradually collected by vacuum space of virtual particles that recycle back to mass in manner of maintaining the quantum conditions of ordinary matter.

Overall, the unification of theory is not that quantum electromagnetic effects equate with gravity; they are only complementary instead as different processes of nature. Gravitational potential increases with mass whereas the electrostatic potential decreases with mass. They do not equate. Unification only pertains to how they interrelate. They equate indirectly by means of an astronomical recycling process and Tired Light mechanism where a Hubble Constant at a distance equal to the nuclear diameter of a hydrogen atom per light speed equals a ratio of gravitational force to electromagnetic force of the hydrogen atom.

ÆTHER AND LAWS OF MOTION

Circular motion was conceived in ancient times as divine, as seemed evident of stars orbiting in the heavenly sky above earthly chaos. Earthly substance was considered as the center of the universe below heavenly stars.

As to explain the primary source of motion itself, Aristotle (384-322 BC) proposed an Unmoved Mover is its provider. What evolved from this proposal was a theory of emanation. Later, for instance, Robert Grosseteste (1168-1253) and Saint Bonaventure (1217-1274) proposed God first created lux, a corporeal form of substance that duplicates itself indefinitely. Motion consists as the duplication of form moving as energy waves in all directions. Lux constitutes the material form of substance by reflecting lumen, which is light. Light is also how God mediates between souls and bodies. As nature takes its course, it is not alienated from God, as He intervenes by emanating light from within.

Light, as according to Aristotelian physics, is thus the essential source of motion, as is distinguished from earthly substance having no inclination whatsoever to move without assistance. This theory was challenged by John Philoponus (about 490-570 AD) in asserting material substance is inclined to remain in motion without assistance. This idea identifies, in part, with inertia: whereby a state of non-acceleration, as mass in relative motion or at rest, maintains unless it is changed by means of an external force, such as gravity or the collision from other mass in relative motion. However, even though it was an insightful idea for advancing theory, it was instead rejected by theologians with more influence in favor of the Aristotelian doctrine.

Light, in modern physics, is still an essential part of mass. The internal energy of mass is $E = mc^2$. The difference of it from Aristotelian physics is the inert aspect of mass allows relative motion to continue, but light is still an internal source of change, and there is further distinction of matter and light to consider. Matter varies in speed from interacting with light or other matter whereas light only varies in speed if moving through material media, such as water or air, or in a gravitational

field. If matter changes speed by its interaction with light, both momentum and energy are conserved according to either classical mechanics or relativity theory.

Impetus and Inertia

A modification of Aristotelian physics did not fall on deaf ears outside of Europe. As during the golden age of Muslim academic culture in Persia, Avicenna (980–1037) concluded motion is an inclination transferred from the thrower that does not cease if it occurs within a vacuum. He obviously realized a decrease in motion requires a resistance to it, such as the presence of air. This reiteration of the Philoponus position identifies with a modern concept of inertia and momentum in view of empty space, but the idea of a plenum was still embedded in general thought. Philoponus and Avicenna, for instance, both conceded that the power of motion given to an object to move through a medium would eventually be used up. Does, then, the light moving through the gravitational fields of the universe surrender its energy to them? It is assumed in this book that it does, as tired light.

Saint Thomas Aquinas (1223-1274) and Francis de Marchia (b. around 1285d. after 1344) also accepted the idea of motion of matter maintaining until it is impeded by the presence of another object, or by gravity, but the idea of matter having innate ability to propel itself forward indefinitely with no additional assistance was granted by Jean Buridan (1295-1358).

Buridan proposed that motion given to an object from another object is sustained by the object until passed onto another object. He named this inherent property of motion impetus. He did not identify impetus with the energy of light, but he did offer a biblical justification for it in interpreting Genesis as stating God rested on the Sabbath after He created the world in six days. Because God rests, He allows His creation to sustain motion such that no longer is there any need for Him to replenish it.

Impetus theory is identical to the modern concept of relative motion except that Buridan referred to rest as distinct from motion, as to allow for an underlying medium such as the æther for a state of absolute rest, as for it to be distinguishable from the relative motion of all matter moving through it. His theory remains consistent with the modern mechanical interpretation of motion insofar as Buridan even stated impetus is proportional to weight times speed. A heavier object or a faster one thus has more impetus, which compares with more momentum in view of modern terminology. Since gravity provides impetus to increase motion towards earthly mass, and earthly mass gives up impetus to escape from earthly mass, a cannonball falling through a hole through Earth is increased in impetus on the way down to the center of Earth the same as the cannonball gives up impetus to move up an equal distance to the surface at the other end. Such analysis later exemplified such periodic motion as the free swing of a pendulum, and of oscillatory motion in general. The latter was to be theoretically developed in the 17th century, but the concept of impetus was interpreted differently by other thinkers in the 14th century. Nicole Oresme (1320-1382) maintained impetus is the temporal quality used up in motion by the inertness of earthly substance tending toward its natural place of rest, the Earth, as Aristotle had contended, and as evident of objects losing motion by falling to the ground. He further distinguished between an impetus given to the motion of the heavenly stars and impetus given to the violent and accidental motion of earthly events. However, he argued, contrary to Aristotle, that it cannot cause an object to accelerate to an infinite speed even in the vacuum of space because the impetus is spent during motion, which is correct if impetus is identified as acceleration.

Oresme further considered Archimedes' principle of the lever whereby the position of a heavier object is placed nearer to the fulcrum for balance. In interpreting this principle as applying to the cosmos, Oresme referred to the Aristotelian idea that earthly substance tends toward the center of the world where Earth rests. However, the moon, sun and other celestial bodies moving about indicated to Oresme the center of gravity could shift. It thus is possible Earth can slightly shift in position as well. He further reasoned, however, that Earth's movement is not provable one way or the other. He further proposed the criteria two equal hypotheses should be the merit of simplicity, as Copernicus later advocated, but Oresme accepted a stationary earth in support of the common interpretation of the Bible at the time.

The Copernican Revolution

While impetus theory developed at Paris, France, more eloquent ways of describing nature developed at Merton College in Oxford, England. Such scholars as John Dumbleton, Richard Swinehead and Thomas Bradwardine proposed an abstract system of degrees and latitudes for analyzing qualities of nature, such as hot and cold, and various forms of motion according to quantity.

The intent of these Oxford scholars was for providing a mathematical description of processes rather than to claim their abstract calculations were actual laws of nature. They nonetheless provided the quantitative means for mathematically analyzing results of experiment in arriving at such concepts as constant acceleration and instantaneous velocity. This is particularly true with Domingo DeSoto (1494/5-1460) applying the calculating technique of the impetus theories developed at Paris to refine the earlier calculations of Oresme for uniform acceleration of falling objects due to constant increase in impetus.

After this development, Nicholas Copernicus (1473-1543) proposed a heliocentric scheme wherefrom the planets, including Earth, revolve around the sun. His scheme was rejected by authoritative rule, but it was later to be advanced by Johannes Kepler (1571-1630).

Aristarchus of Samos (b. around 310 BC-d. about 230 BC) advocated a heliocentric system of planets, including Earth, revolving around the sun. However, his scheme was overcome by the greater influence of Aristotle, who did not recognize the relatively great distance of stars whereby there is a condition of parallax.

Parallax is the apparent change in position of an object caused by the observer moving here and there. The positional change appears greater for closer objects. Nearby scenery, for instance, changes rapidly for passengers looking out the window while inside a moving automobile, whereas a more gradual change in relative position occurs of more distant mountains. The change in the relative position of the sun appears not to change except for Earth rotating for day and night to occur. Earth revolves around the sun, as well, for a change of seasons. Such slight change can either be interpreted as parallax of Earth's motion or the sun's motion. Description of the latter is sometimes more complicated. A complex system of epicycles, for instance, was used for describing circular paths of planets and sun revolving around Earth.

Copernicus did not consider parallax. Apart from it, observations of astronomers provided more and more data on the relatively closer positions of planets in our own solar system, such that the scheme of circles within circles for planets moving around Earth was more complicated. Copernicus thus proposed the much simpler scheme of Earth circling the sun. He also asserted no internal effects of Earth moving through space are detectable since all relative parts move uniformly, which implies a principle of relative motion.

Publication of Copernicus's book *De Revolutionbus* occurred the same year of his death, but the book was outlawed along with later works by any of its defenders, including Galileo. Giordino Bruno (1548-1600) advocated, for instance, an entire world full of solar systems, and he further speculated stars move relative to each other, but that they are too distant from us for detection of their relative motion. However, he was burned at the stake for his outspokenness, whereas Galileo was merely sentenced to confinement for his defiance of the order.

Copernicus had nonetheless set forth a revolution in thought. Readers in Italy and elsewhere in Europe accepted *De Revolutionbus*. Simon Steven (1548-1620) of the Netherlands supported the heliocentric system with his book *De Hemelop* he published as early as 1608. However, the Copernican system was not faultless, as celestial data compiled by astronomers indicated planetary motion was not of true circles. As Copernicus revered the circle as a divine principle, his scheme included thirty-four epicycles. This weakened his argument of simplicity. A Copernican revolution had nonetheless begun. Ironically, Johannes Kepler, a so-called mystic who claimed to listen with a sensitive ear to the musical harmony of planets in motion, including Earth, was to defend it.

Kepler's Celestial Scheme

Kepler (1571-1630) was proficient in mathematics. Such skill enabled him to become an assistant to Tycho Brahe's (1546-1601) in plotting data of celestial movement. Brahe had rejected the heliocentric system in favor of the stationary Earth mainly because he could not detect parallax of the stars. He regarded Earth material as an inert condition of nature. However, he did accept a heliocentric system as applying for all the planets other than Earth revolving around the sun. As he maintained the sun and moon only revolve about Earth, it would only be another step to include Earth as also revolving around the sun.

Kepler studied the data compiled by Brahe and in 1609 proposed three empirical laws to describe it: 1. Planets move in elliptical paths. 2. An equal area in equal time is swept between the sun and a planet. 3. Orbital periods squared in ratio to the cube of the planet's average distance from the sun are the same for every planet.



In view of the first law, an ellipse differs from a circle in that it has two foci in place of a center. A property of an ellipse is a total distance between any two straight lines connecting the two foci to anywhere on the perimeter of the ellipse is always the same. As illustrated above, consider a pencil maintains a taut string between two locations for drawing an ellipse. The excess length of the string between the two loci determines the eccentricity of the ellipse such that the total length of the two upper lines in the above illustration equals the total length of the two lower lines, which is also a twice average distance of each focus from the perimeter to represent a semi-major axis comparable to the radius of a circle.



The second law refers to the distance between the sun and a planet. The focus position of the sun is more massive than focus positions of planets for the planets to orbit around the sun. The second law states the area swept per time between the sun and planet by the planet is the product of the distance of the planet from the sun and the speed of the planet at its distance from the sun. The planets thus move faster when closer to the sun for them to maintain the same amount of area per time, as illustrated above. (A circle is an ellipse with the foci in the same position. The average length of the two foci equal the radius of the circle, and orbital speed of a circle is the average orbital speed of the ellipse. Because the twice radius has four times more area for a perimeter of twice distance, the speed reduced by upward deceleration to half as much speed thus takes four times longer to complete the circle at half speed, equaling the same area.)

The ellipses are not the same for each planet, but Kepler determined a common property as his third law. By it, a planet's orbital-period-squared in ratio to the cube of the planet's semi-major-axis as its average distance from the sun is the same for all planets. For simplicity of calculation, consider an Earth's orbital period to be one year. If its average distance from the sun is also determined as one unit (AU), chosen as such, its orbital-period-squared in ratio to its semi-major-axis-cubed is one-numerical-unit, which allows for a simpler means of comparing the ratios of the other planets with Earth's.

The average orbital distance is that of the semi-major-axis. The orbital period is orbital distance per orbital speed. Both distance and speed of orbit determine the orbital period. A planet with a semi-major-axis that is twice that of Earth's has twice the orbital distance while moving at a slower rate. If slower by the square root of 2, its orbital period is 2 (as according to the longer distance) multiplied by the square root of 2 (according to the slower speed). A period squared is the square of 2 multiplied by the square of the square root of 2: $(2\sqrt{2})^2 = 8$. The twice more distance of a semi-major-axis cubed calculates as $2^3 = 8$, and 8 divided by 8 is also unity, the same ratio of Earth's period squared, and semi-major-axis cubed as unity.

It is possible Kepler could have had insight of the laws of nature, but they were developed in a progressive manner by other legends of history as Galileo and Newton. According to Newton's inverse square law of gravity, for instance, the average orbital speed v relates average orbital distance r as centripetal acceleration: v^2/r . Orbital speed around the same mass at twice distance decreases by the square root of 1/2, such that its period per distance increases by the square-root of 2. Twice orbital-period-squared multiplied by twice an orbital distance is similarly 8 times greater, as is twice the orbital-radius-cubed. Newton's inverse square law thus equates the same as does Kepler's third law and explains the square root condition of speed.

Terrestrial Mechanics

Physics refers to the laws of motion as mechanics. Complementing the celestial mechanics of the planets is terrestrial mechanics. Its development also connects with gravity, as with determining the nature of bodies falling to Earth.

An experiment on gravity was done in the third century BC by Strato in determining sounds of falling bodies hitting ground differ for a different height of fall. He thus surmised an increase in speed occurs during fall due to the louder sound of impact of an object's fall from a greater height.

An experiment in the thirteenth century was an attempt by Jordanus de Nemore (1225-1260) to distinguish weights of objects according to their angles of decent while sliding along planes. His theory of positional gravity and component forces considered work in relation to the position of a level apart from where it balances in a state of equilibrium.

Leonardo da Vinci (1452-1519), whose work did not all survive except for his notes, attempted to determine if the gravitational fall of an object is directly towards the center of Earth. He dropped two heavy objects from a tower in a failed attempt to find a decreased distance of separation.

Da Vinci failed to determine the direction of fall because the change in distance is too minute to detect with the instruments he had available, but he discovered instead a pyramidal increased speed of fall in equal intervals of time in analogy to counting stairs. However, from his notes, which might not necessarily reflect what he actually concluded, he incorrectly stated the distance of fall is proportional to time instead of time-squared.

Oresme, Galileo and others correctly determined the distance of fall is proportional to its time squared, but da Vinci might have only erred in his taking notes of his findings. To his credit, da Vinci seems to have been wise to the ways of nature. For instance, in anticipation of Newton's third law of motion, whereby force and the resistance to force are mutually the same, he suggested air and water counteract with the same amount of force.

Demonstratio, published in 1552 by Giovani Battista Benedetti (1530-1590), was a book that attempted to determine the nature of falling bodies analytically. He first assumed bodies of different weight fall at the same rate if they are equal in density, as composed of the same material. His proposal was contrary to Aristotle's doctrine that a heavier body falls faster than does a lighter one, but in a revised edition published in 1554 Benedetti changed his position to that of bodies of the same material but of different size do not fall at the same rate.

Perhaps critics influenced Benedetti to change his position, or perhaps geometrical considerations were apparent. If an object divides into two or more parts, for instance, more inside becomes part of the outside, which is also more exposed to atmospheric conditions. Since only the surface area changes instead of total volume of all its parts, this means the mathematical ratio of volume to its surface area is relatively according to its size. Smaller objects thus encounter more friction per surface area to volume or weight, which causes them to move more slowly through their medium, as is true of smaller particles falling through the atmosphere.

The difference in size of bodies falling at different rates is indicative of a medium. They fall at different rates in a medium, but they fall at the same rate in vacuum space. However, this is only true of such media as water and air. A medium with small enough parts even great in number could saturate between material parts. Thus, if internal components of atoms are the same size and density with regard to the permeability of the medium that they are moving through, then it affects them all equally. As it were, for supposedly not knowing the true nature of atomic particles, Galileo Galilei (1564-1642) theorized instead that all bodies (regardless of their size, weight or material they are comprised of) fall, in vacuum, at the same rate. Note: Magnetic permissibility and electric permittivity are measurable effects in relation to space free of mass, whereby light speed in free space is per the product of permittivity and permissibility squared of free space. Not determined is whether totally gravitationally free space actually exists.

The abstract analysis of motion at Merton College likely guided Galileo rather than the works of Benedetti. In any case, the hypothesis Galileo put forth provided a means of testing whether the laws of motion typify objects as moving through empty space, as would seem necessary for the planets in the Copernican system to move unopposed by a medium. Galileo therefore experimented with objects moving along planes and took notice of the free swing of pendulums to discover an appreciable amount of reduced friction tends to allow motion to maintain. He thus postulated the first two laws of motion that Newton would later formulate in his system of mechanics with regard to the inertia of continual motion and its acceleration as a change in either speed or direction, or both.

In order to test the equality of fall between masses, it is likely Galileo performed experiments, as is alleged of his dropping objects off the leaning tower of Pisa, but such a claim is uncertain. Nicole Cabeo (1585-1650) had conducted experiments in 1641 to confirm his claim that objects do not fall at the same rate. When informed of the experimental results, Galileo replied how difficult it is to attain accurate results from such experiments.

Galileo seemed to take other results of experiment for granted as well. For instance, Pierre Gassendi (1592-1665) had directed an experiment to be performed on a moving ship at sea to find out if an object follows a straight course of the ship while falling to the foot of the mast. His own experiments having already verified his laws of motion, Galileo confidently asserted the object would fall the same as if the ship did not move.

Still, to his credit, Galileo truly deserves the acclaim of pioneering the modern approach of establishing laws of nature according to observational facts rather than by ontology or abstract concepts of intuition.

Weighing the Debate

The idea all of space is filled with an undetectable medium is contrary to the empirical approach, but the new discoveries do not prove there is an empty space to move; it only affirms objects move as though empty space is before them instead of that it actually exists before them. Moreover, the new mechanics is not a complete explanation of reality. It does not explain, for instance, how it is possible for corporeal matter to attain and maintain a form in the manner it does. How, for instance, are elastic collisions possible without an internal force to maintain material form?

Such questions were still being pondered. Francis Bacon (1561-1626), for instance, questioned such concepts as a vacuum state with regard to the nature of matter. If matter consists of individual atoms, then how are they kept intact? He concluded atoms need to somehow possess inner qualities by means of some intangible spirits arising from a medium of some sort to provide cohesion and form. He also advocated a primary role of science is to describe nature according to how it is observed.

Another philosopher who did uphold the atomic theory as an internal mechanism was Gassendi. He offered an atomic theory in view of primary and secondary effects. Secondary effects are of inertia and motion. Inertia is necessary to resist penetration and to change the motion of other atoms by means of direct contact. The atoms sometimes combine to produce various effects, such as for our observable world to be comprised of the secondary effects arising from a primary source that produces the form and cohesion of the secondary effects. The primary source is visibly indeterminable by us even though it gives rise to the secondary effects that actually constitute the natural world of observation.

A philosopher who went so far as to advocate a plenum in view of the concepts of relative motion and inertia was Rene Descartes (1596-1650). He was well aware of the implications of these new concepts, but he opposed the vacuum state. He thus undertook the task of explaining relative motion and such effects as gravity in view of a plenum.

Since motion through a plenum is by reason necessarily circular, the Cartesian universe contains vortexes that differ in size and rotational speed. Exchanges occur by smaller invisible vortexes accelerating to greater speeds in determining the weight of heavier ones, and the visibility of the world is determined by the size of our nerves extending from our brains. Our seeing as humans is, in fact, dependent on how our brains can comprehend all the many images our eyes allow it to focus on.

Descartes further postulated conservation of motion, which is similar to conservation of momentum in that in a collision between two masses the change in speed of the greater mass is less than the change in speed of the lesser mass. Twice an increase in speed of one half as much mass nullifies a half decrease in speed of the twice mass to thus conserve total motion.

As with regard to inelastic collision, motion can be conserved with the creation of heat either as the internal motion of molecules or as radiation, as is light.

It is also possible that radiation can evade the senses. In this case, it is possible that motion is conserved in the observable world in the form of a potential detectable by our senses in contrast to its underlying mechanism. In this regard it seems possible matter can somehow break apart in a way it evades our senses for empirical detection. In order to maintain conservation of motion, as momentum and energy, the invisible medium absorbing mass need be affected such that it forms new matter in place of lost matter.

Modern theory contains similar ideas, as with regard to virtual particles theorized according to a condition of probability with no effects as causally determinable. Explaining effects of the natural world is thus according to a virtual field, which could also be used to explain gravity by means of virtual particles moving undetected away from matter to create vacuum effects for attraction.

Mass and Volume Ratios

It is proposed that our observable world creates from a plenum of no variable density. Since protons and electrons have the same plenum content per space, a mass-energy of that content is connected with the surrounding space whereby the volume ratios need to coincide with mass ratios. Carl R. Littmann and Greg Volk, among others, have investigated the similarity of volume ratios to mass ratios. Such atomic masses as the proton, kaon, pion and muon are indeed typical of this primary process.

One of Littmann's simplest volume ratios with mass ratios is that with regard to the pion mass in ratio to the electron mass. As depicted below, if three large spheres circumscribe a smaller sphere, as in a plane, the radius of the large sphere is 6.464 times longer than the small sphere radius, and the volume of the large sphere is 270.1 times larger than the small one. A pion mass on the average, as with positive, negative or neutral charge, is 270.13 times the electron mass.



The kaon-electron mass ratio comparison, as illustrated below on the left, is three large spheres circumscribing three small ones, whereby a large sphere radius in ratio to an inner sphere radius is 9.89898 to 1, whereby their volume ratio is 969.99912 to 1. The average between the kaon mass either charged or not charged in ratio to the electron mass is 969.98.



The proton relates as an average of two volumes, one of the three small spheres packed between crevasses of three large spheres and the other of three small spheres positioned opposite the crevasses. As for the small spheres packed between crevasses, the big and small sphere radii ratios relate as the kaon, but if the small spheres are positioned opposite the crevasses, the radii and volume ratios are 13.9282 to 1 and 2702 to 1. The average of 970 + 2702 as 1836 is comparable to the proton mass of 1836.15 electron masses

The muons below compare to the proton in the manner of the average size of two large circles circumscribing smaller circles. Circumscribed are two same size circles in one large circle, and three same sized smaller circles within a large circle. The large circles compare to a proton mass of 1836.15 electron masses. The ratio of radii between 1 of 2 small circles to its large one is $\frac{1}{2}$. The ratio of 1 of 3 small circles to its large one is $\frac{1}{8}$ and 0.0999619 to 1, respectively. An average of the different volumes of the two smaller circles in ratio to the larger one of the 1836.5 electrons is a ratio of 206.53 to 1836.15. The empirical value of the muon is determined as 206.7682838(24) electron masses.



In view of the volume ratios in relation to the proton, kaon and muon, it is conceivable that a muon of 206.77 electron masses is ejected along with an electron and light energy from 3879.92 total electron masses consisting of 2 non-charged kaons and 2 charged kaons in the formation of a proton and neutron totaling 3674.83 electron masses. Four kaons minus the muon and an electron are twice 1836.12 electron masses.

The number 9.89898 as the ratio of a kaon radius to an electron radius is significant in that it has been derived separately by Greg Volk and Harold Aspden. Volk related it in also comparing mass ratios to volume ratios, but Aspden derived 8.89898, which is (9.89898 - 1), according to a combination of electrostatic formulas in relation to the creation of protons from muons. An electron volume and mass as one unit implies a loss of an electron.

Greg Volk calculated the number 9.89897 according to the tetrahedral pattern of 4 spheres packed around a common origin such that the surface of a sphere centered at the origin and touching centers of the four spheres calculates as coordinate distance points $(1 \ 1 \ 1) \ (1 - 1 - 1) \ (-1 \ 1 - 1) \ (-1 \ -1 \ 1)$ from 0 as distance **D** according to three perpendicular axes as an extension of the Pythagorean Theorem for three-dimensional space. **D** thus calculates as the square root of 3, 3 being the sum of each coordinate length squared. In contrast is the radius **R** of any one of the other four spheres. Since they are symmetrically aligned along planes of respective axes, **R** calculates as the square root of 2, 2 being the sum of each coordinate length squared to the plane.

The ratio of $(\mathbf{D} + \mathbf{R})$ to $(\mathbf{D} - \mathbf{R})$ calculates as

$$\frac{D+R}{D-R} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = 9.898979486$$

It equals Littmann's radii ratio of the three large spheres circumscribing the three smaller spheres. It is also exactly one number greater than a particular factor used in relating muons to the ratio of proton mass to electron mass.

Harold Aspden (1927-2011) derived the number above minus 1 in an attempt to explain the ratio of proton mass m_p to electron mass m_e in ratio to energy of a ubiquitous muon m_{μ} to electron energy in the manner

$$\frac{m_p c^2}{m_e c^2} = \frac{\left\{9 - 2\left[\sqrt{\frac{3}{2}} - 1\right]^2\right\}m_\mu c^2}{m_e c^2} = (8.8989795)(206.3329) = 1836.1522$$

An empirical value of m_p/m_e as1836.1527 compares favorably to Aspden's 1836.1522, but his m_{μ}/m_e value of 206.3329 is slightly off the mark of the empirical value of 206.7683. However, the muon in the above formula is an even more ghostly muon subject to virtual effects. Aspden had also, to his credit, derived a value of 206.768038 as well as having predicted values of a proton-electron mass ratio, a fine structure constant and proton's magnetic moment nearly accurate to their values as presently determined.

The number 9.898979486 that Littmann and Volk used in relating the radii of the electron and kaon is one electron more than a proton and muon ratio of masses according to Aspden's formula. Aspden's derived formula is according to a combination of two formulas previously derived by Charles Augustin Coulomb (1736-1806) for internal action between particles and by Joseph John Thomson (1856-1940) relating to internal energy of particles. They are, respectively, of the forms

$$\frac{-e^2}{(x+y)^2} = -mv^2 \qquad \frac{2e^2}{3x^2} = mc^2$$

The Coulomb formula is according to electrostatic interaction between two particles of opposite polarity whose centers are separated at a distance equal to (x + y). Thomson's formula pertains to internal mass-energy of mass m of radius x. The Coulomb force is attractive, as negative, and assumed here as containing the internal mass-energy m within the sphere of radius x. The two forces are assumed equal for a value of v as c, such that

$$\frac{-e^2}{(x+y)^2} + \frac{2e^2}{3x^2} = 0$$
$$\frac{e^2}{(x+y)^2} = \frac{2e^2}{3x^2}$$
$$\frac{e^2}{e^2} = \frac{2(x+y)^2}{3x^2}$$
$$\frac{3}{2} = \frac{(x+y)^2}{x^2}$$
$$\sqrt{\frac{3}{2}} = \frac{x+y}{x} = 1 + \frac{y}{x}$$
$$\sqrt{\frac{3}{2}} - 1 = \frac{y}{x}$$

Assuming the proton mass equals 9 ubiquitous muons minus twice the ratio y/x squared, then $m_p/m_e = (9 - 2y^2/x^2)(m_\mu/m_e)$.

Generally, the proton is contained within the nucleus of the atom that has a radius that is about 1836 times shorter than the atom. The actual radii of the electron and proton are theoretical. The plenum itself does not vary in density, but it is reasonable that particles of higher energy are contained within a smaller space from a larger space, such that the larger space is both the medium of their creation and their containment.

Such speculation on mass-volume ratios might not have any practical value, but tetrahedron and icosahedron patterns have proven to be useful in chemistry and biology. The icosahedron is used in chemistry for describing Nanoparticles contained in crystal cluster compounds of boron and carbon atoms. In biology, particular viruses protected by protein surfaces are found to contain subunits that also best fit the icosahedron.

NEWTONIAN MECHANICS

Although Isaac Newton (1642-1727) is considered the founder of Classical Newtonian Mechanics, there is nearly no part of it, if any, that had not been thought of by someone else. His first two laws of motion, as with regard to momentum and force, are attributable to the works of Buridan and Galileo. John Wallis (1616-1703) stated the second law in 1603 and the third law of motion, with regard to mutual action and reaction, was similarity offered by Leonardo da Vinci (1457-1515) in claiming air and water equally resist each other. Robert Hooke (1635-1703) claimed he had suggested to Newton the inverse-square-law of gravity, which Newton then formulate according to Kepler's planetary laws of motion.

Newton stands out nonetheless as outstanding for his contribution in advancing theory. It was a comprehensive formulation of ideas that resulted in a unification of Kepler's heliocentric scheme of the solar system in which relative motion and gravity equate as forces of nature.

Laws of Motion

In addition to the concepts of absolute space and absolute time as the means to determine events according to standard units of measure, Newton believed the material content of the universe always stays the same. It does not change by being in relative motion, under the influence of gravity, or by any means whatsoever. From this conservation of mass and assuming space and time are absolutes, he postulated three laws of motion:

- 1. Law of inertia: objects in a non-accelerating state of relative motion or at rest remain as such until they are acted on by an external force, such as by either gravity or collision with other mass
- 2. Force is the product of a mass m and its acceleration *a* per time t with regard to acceleration
- 3. An equal and opposite reaction occurs with every action.

The law of inertia is expressed as the product of a mass m and its velocity v as momentum P. Hence, the first law is according to the equation P = mv. By this law, momentum remains unchanged until acted on by another mass or external force.

With regard to the second law, the amount of force F used to change momentum is the product of mass m and acceleration *a*, as according to the equation F = ma. Acceleration itself is either a change in velocity, as either of speed or of direction, or of both, per time.

A change in velocity per time determines acceleration *a* such that it is possible a quick enough fly exerts more force from changing speed from at rest to fifteen miles an hour than does someone who throws heavy brick twenty miles per hour. However, since the brick has far more mass than the fly, more force is generally applied to it rather than to the acceleration of a fly. More definitively, force is per time and per mass. The same change in a speed of twice as much mass in twice the time is the same amount of force, as the lesser mass changes at twice the rate:

$$F_1 = \frac{m(\Delta v)}{\Delta t} = F_2 = \frac{2m(\Delta v)}{2\Delta t}$$

The Greek delta letter Δ denotes change, as change in velocity from $v_0 = 0$ to v_1 , and as change in time from say $t_0 = 0$ to t_1 as one second.

Since velocity includes both direction and speed, the change in velocity can either be a change in speed or a change in direction, or a change in both direction and speed. A rocket moving in a circle, for instance, constantly accelerates by means of constant force. The rocket can also constantly increase in speed by more force increasing the rate of circular acceleration.

Galileo had previously established these first two laws. Newton added the third one: the law of mutual force. Hence, if a force acts on a mass, the mass reacts with an equal and opposite force for a change in momentum of the rocket caused by the rocket fuel to result in an equal amount of change in momentum of the rocket fuel in the opposite direction, as the rocket fuel would otherwise be inexhaustible.

From the mutual action and reaction between masses is conservation of momentum. Conservation means staying the same, and conservation of momentum means a total momentum of all mass in any particular direction never changes by the action of one mass on another. The action can either be a collision of two or more masses or a force such as gravity. If a greater mass collides with a lesser mass, a conservation of the total momentum of the action is maintained by the change in velocity of the greater mass in one direction being relatively small and the change in velocity of the lesser mass in the opposite direction being relatively large. The mutual changes in each of the momentums is according to the equation

 $M(\Delta v) = m(\Delta V)$

Change in velocity of the greater mass is thus less than the smaller mass.



Consider our moon orbiting Earth as an example of equal changes in momentums of masses caused by their gravitational influence on each other as mutual. In this case, a relatively slower moving Earth being more massive than the moon results in a change in direction of the moon being a greater change in velocity at a greater speed. The moon's orbit around Earth is thus a relatively large circular path while Earth orbits within a small circular path of the larger one. As to why the Earth's path does not circle the moon, it is because Earth is too slow changing its direction at each new position of the moon.

Centripetal Acceleration

Centripetal acceleration is a constant change in direction resulting in a circular path. In 1966, Newton formulated it mathematically, but he delayed the publication of his work for many years. Christian Huygens formulated it independently for his publication of it in 1673, but it was Newton who used it for the unification of celestial and terrestrial mechanics.

Consider, as by Newton's first two laws of motion, a particle tends to move in a straight path, but it actually moves peripherally instead around a fixed point because of a centripetal force constantly acting on the particle in changing its direction. The centripetal force is any agent, such as gravity or whatever, preventing the particle from escaping its orbit. The mathematical expression is

$$a = \frac{v^2}{r}$$

Respectively, the letters *a*, r and v represent the average rate of acceleration, orbital radius and orbital speed.

Although centripetal acceleration is a particular form of acceleration, it equates to any form of acceleration generally expressed by the equation

$$a = \frac{\Delta v}{\Delta t}$$

Acceleration is thus generally a change in velocity (as speed or direction or both) per change in time.

As illustrated below, a system moves along the arc at constant speed v in time T from initial velocity v_i to a final velocity v_f in relation to triangles. Radius r_1 , an extended tangent of r_1 to the right, and a radius r_2 extended to the tangent of r_1 form a right triangle. Another right triangle forms from the extension of the tangent of r_2 to the tangent of r_1 . Because of the common vertex at the right, it is a smaller upside down and similar-right-triangle. All ratios of the corresponding parts thus equate. Because distance results from duration of speed, the vector directions v_i and v_f are representative of both distance and velocity. The other leg of triangle with legs v_i and v_f is a vector direction pointing towards the center of the circle to represent a change in velocity, and it is thus denoted as Δv . The ratio of $r_2 + \Delta v$ and $x + v_i$ of the larger right triangle is the same ratio of the smaller one, Δv and v_f , such that

$$\frac{x + v_i}{r_1} = \frac{\Delta v}{v_f}$$

The values x, v_i and v_f are interpreted as either speed or distance for $x + v_i$ and v_f to relate respectively as v and vT, where T is the time of acceleration.



These relations are according to a smaller distance during a less time of acceleration. For the smallest possible angle between radii, the arc between r_1 and r_2 converges with line segments s and v_f at the limit for shortest time of acceleration to equate in the manner

$$\frac{\Delta v}{v\Delta T} = \frac{v}{r}$$
$$a = \frac{\Delta v}{\Delta T} = \frac{v^2}{r}$$

Centripetal acceleration thus equals a constant change in direction towards the center of the circle.

Kepler's Scheme

Since force is defined as F = ma, centripetal force similarly equates as

$$F_{c} = \frac{mv^{2}}{r}$$

Note: F_c increases for smaller r, as for faster change in direction at the same speed around a smaller circle.

Another nature of force is gravitational, which is also centripetal with regard to it maintaining orbital motion. This connection was the means that enabled Newton to unify Kepler's celestial scheme with the forces of nature for the formulation of a theory of gravity.

Using Kepler's planetary laws of motion, Newton derived the inverse square law for gravity. Kepler's third law, in particular, relates the planetary orbits in our solar system. Accordingly, the period squared of any elliptical orbit equals the cube of the mean distance of the planet from the sun. The ratio of the time-squared of Earth's revolution around the sun to a cube of its distance from the sun (as from Earth's center of mass to the sun's center of mass) is thus the same as that of any other planet in the solar system.

Kepler's third law in mathematical terms is

$$A^2 = kr^3$$

The letter k represents a constant of proportionality for the proportionality between the period of revolution A-squared and the mean radius r-cubed of the semi-majoraxis of an elliptical orbit.

If the ellipse is a circle, which it can be, then r is the radius of a circle. As for simplicity, let the distance of orbit be that along the circumference of a circle such that the orbital distance is π times twice the radius of the circle. (An average length of the two foci of an ellipse equate as a radius of a circle, such that a circle is truly representative of the average distance of an ellipse, whereby the two foci have converged to the same position.)

The time or period of revolution of the planet can also be expressed in terms of distance divided by average orbital speed in the manner

$$A = \frac{2\pi r}{v}$$

Squaring and combining equations gives

$$A^2 = \frac{4\pi^2 r^2}{v^2} = kr^3$$

Multiplying the last two sides of the previous equation by v^4 , and by dividing it by k and r^4 , gives

$$\frac{v^2}{r} = \frac{4\pi^2}{kr^3}$$

Centripetal acceleration according to Kepler's scheme is thus proportional to $4\pi^2/kr^3$.

Multiplying both sides by m relates to centripetal force in the manner

$$F_c = \frac{mv^2}{r} = \frac{4\pi^2 m}{kr}$$

According to Newton's third law of motion, the gravitational force between two bodies of mass is the same. Since r is a common distance separating the combined forces of masses m_1 and m_2 , they combine by multiplication for the total force to equate in the manner

$$F_g = \frac{4\pi^2 m_1}{kr} \cdot \frac{4\pi^2 m_2}{kr} = \frac{16\pi^4 m_1 m_2}{k^2 r^2} = \frac{Gm_1 m_2}{r^2}$$

This equation expresses Newton's general form of the inverse-square-law of gravity, with G as a constant of proportionality in place of $16\pi^4/k^2$. Its determined value is 6.67428(27) x 10⁻⁸ cubic centimeters per grams and per seconds squared.

A Galilean Interpretation

Newton derivation of gravity can also be shown to be consistent with Galileo's discovery that all bodies of mass gravitate at the same rate.



Galileo determining that all mass gravitate the same rate towards Earth indicates gravitational force from each part of Earth is in proportion to the amount of mass it gravitates. Gravitational action of Earth thus doubles for it to act on twice as much mass. However, there is also an internal action of twice mass interacting with itself and the calculation depends on how mass divides as individual parts. As
illustrated above, simply divide an Earth and moon each into an equal number of parts and count the number of mutual attraction between them.

This is simple multiplication. Total interaction of gravity between the Earth and moon is simply a number of moon parts multiplied by a number of Earth parts. It relates to Newton's inverse square law in the manner

$$\frac{GM_1m_1}{r^2} = \frac{M_1v_1^2}{r} = \frac{m_1v_2^2}{r}$$
$$\frac{G(M_3)(m_3)}{r^2} = \frac{(3M_1)(3v_1^2)}{r} = \frac{(3m_1)(3v_2^2)}{r}$$

Three times mass thus equates to nine times centripetal force even though the speeds of centripetal acceleration only increase by the square root of 3.

Further analysis is with regard to density and size. Consider twice mass at twice radius. Since it has eight times more volume with twice the mass, its density is onefourth in comparison. It has the same Newtonian Potential; such that, its rotational acceleration and centripetal force at twice radius and same speed is also the same. The reason it is the same is because half as much change in direction is counterbalanced by twice as much mass changing direction. The one-fourth force at twice radius is counterbalanced as well by twice as many particles interacting four times as much, which further indicates a cause of gravity by means of an exchange of energy between particles. Gravity could very well occur by means of vacuum effect in the wake of emitted radiation caused by this interaction between particles.

Explaining Gravity

Newton was not content with his inverse-square-law for gravity being only capable of explaining action of one mass on another as occurring at a distance. He considered such action at a distance as casually absurd, as he attempted to explain gravity more completely in the manner of a contiguous action of masses affecting the space between them, as for action between masses to somehow result in their mutual attractions.

Although Newton attempted to explain gravity according to an agent acting between mass for contiguous action to occur, he was still reluctant to recognize an ather filled space as the medium for wave action. He regarded its presence as an obstacle to the free movement of planets and the natural motion of mass in general unless it could be rare in content. He considered an ather comprised of extremely rapid moving particles of minute mass for internal elasticity of containment. In theory, a particle of less mass can have a greater speed while it moves with the same momentum as does the more massive, slower moving particle, such that a containment of the former can allow less resistance to movement of other mass than would a containment of the latter. Æther is thus allowed to exist only as nearly massless particles of extremely rapid, elastic collisions. As nearly massless, they allow ordinary particles to move in a path relatively free of content before them.

Newton's æther has merit and possible truth inasmuch as gravity could very well be the result of a vacuum effect from the interaction of æther with mass. Since mass accelerates speeds of lesser inert æther particles, the faster particles are more inclined to escape, as for leaving a vacuum effect in their wake. However, escaping particles need to somehow be replaced in order for the process to maintain continuance.

Kinematics

An escape velocity ve derives from kinetic energy in the manner

$$\frac{1}{2}mv_e^2 = \frac{GMm}{r} = mv_o^2$$
$$v_e = \sqrt{\frac{2GM}{r}} = v_o\sqrt{2}$$

The mass m can thus overcome the gravitational binding energy of relative motion if the speed v_e of escape is at least the square root of 2 times more than the orbital speed v_o at radius r from the center of mass M.

Although force is energy, in Newtonian Mechanics it more specifically applies to change in speeds or to the resistance of such change. Energy is a word used to describe such action as its latent potential.

Kinetic derives from the Greek word kinesis that means motion, but it was not applied until a much later time, as about 1850 by Lord Kelvin.

Gottfried Leibniz (1646-1716) called the energy of motion "Vis viva", meaning the living force. Willem's Jacob Gravesande (1688-1742) found the penetration of clay by equal weights that are dropped from different heights is in proportion to the difference in speeds squared of each weight. Emilie du Chatelet (1706-1749) explained this result as work energy of the change in speed of mass used to move a mass quantity the distance proportional to the inertial resistance causing the change in speed.

With calculus, Joseph Lagrange (1736-1813) found in 1811 there is an additional difference of Vis viva mv² and a potential energy of momentum mv by a factor of 2. A factor $\frac{1}{2}$ for kinetic energy K = ($\frac{1}{2}$)mv² came from Gustave Coriolis (1782-1843) in 1829, even though a kinetic theory of gases was developed earlier by Johann Bernoulli (1667-1748) in accordance with Newton's laws of motion.

Consider a moveable partition separating two gases into equal volumes of cubic space. Each gas has the same number of molecules, but molecules of one gas have only one fourth the mass moving at twice the speed, on the average, than does the more massive molecules of the other gas. Since $(\frac{1}{2})m(2v)^2 = (\frac{1}{2})(4m)v^2$, the gases have the same kinetic energy, and since a one-fourth mass at twice the speed strikes the partition twice as often, the momentum of action on both sides of the partition per time of action is the same, such that the partition does not move.

Conservation of kinetic energy of elastic collision is proven with three equations according to Newton's laws of motion:

(1a) $(nm)v_1 + mv_2 = (mn)v_3 + mv_4$

(2a)
$$\frac{1}{2}(nm)v_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}(nm)v_3^2 + \frac{1}{2}mv_4^2$$

(3a)
$$v_1 - v_2 = v_4 - v_3$$

Accordingly, n denotes any positive real number such that the product nm denotes any mass quantity as proportional to m. The left sides of equations (1a) and (2a) represent the momentums and kinetic energies of the masses before collision. The right sides of these equations represent the same after collision: the velocities v_1 and v_2 are before elastic collision, and velocities v_3 and v_4 are after collision. Equation (3a) defines elastic collision such that the difference in relative speed between masses remains the same after collision as before collision, but they have been reversed in their original directions.

Equations (1a) and (2a) are divided by m, and both sides of equation (2a) are multiplied by 2 for simplification, obtaining

- (1b) $nv_1 + v_2 = nv_3 + v_4$
- (2b) $nv_1^2 + v_2^2 = nv_3^2 + v_4^2$
- (3b) $v_1 v_2 = v_4 v_3$

To prove kinetic energy is conserved in elastic collision, the task is to derive equation (2a) from equations (1b) and (3b).

Equation (1b) rearranges by subtracting v_2 and nv_3 from both sides of it to obtain

$$nv_1 - nv_3 = v_4 - v_2$$
$$n(v_1 - v_3) = v_4 - v_2$$

By dividing both sides of the resulting equation by $(v_1 - v_3)$ a solution of n is obtained as

$$n = \frac{v_4 - v_2}{v_1 - v_3}$$

Rearranging equation (3b) by adding v_2 and v_3 to both sides of it obtains

$$v_1 + v_3 = v_4 + v_2$$

Dividing both sides of it by $(v_1 + v_3)$ obtains

$$1 = \frac{v_4 + v_2}{v_1 + v_3}$$

The product of the two solutions n and (1) gives

$$(1)n = \left[\frac{v_4 - v_2}{v_1 - v_3}\right] \cdot \left[\frac{v_4 + v_2}{v_1 + v_3}\right] = \frac{v_4^2 - v_2^2}{v_1^2 - v_3^2}$$

Multiplying the first and last equalities of this equation by $(v_1^2 - v_3^2)$ obtains

$$n(v_1^2 - v_3^2) = v_4^2 - v_2^2$$
$$nv_1^2 - nv_3^2 = v_4^2 - v_2^2$$

Adding $nv_3^2 + v_2^2$ to both sides of this result obtains

$$nv_1^2 + v_2^2 = nv_3^2 + v_4^2$$

Multiplying both sides by m obtains

$$(mn)v_1^2 + mv_2^2 = (mn)v_3^2 + mv_4^2$$

(2a) is thereby derived from (1a) and (3a) in proving conservation of kinetic energy in elastic collision from the laws of conservation of momentum and conservation of the difference in relative speeds after collision.

Conservation of kinetic energy is maintaining relative motion between mass by elastic collision. Twice mass changes the relative speed of the other mass twice as much by decreasing its relative speed half as much. The total increase in speed of mass is thus the same as the total decrease in speed of mass. Contrary to this analysis is inelastic collision whereby relative motion between mass appears to decrease. However, an inelastic collision is merely more complex, involving such internal motion as the creation of heat or the emission of electromagnetic radiation for more complex analyses.

KINEMATICS ATOMS AND THERMODYNAMICS

Sometime in the thirteenth century Giles of Rome (b. before 1247-d. 1316) proposed an atomic theory according to a condition that no form of matter exists smaller than a minimal quantity of substance. He tried to support his theory with the investigation of a vacuum state. He was unable to verify his theory, but investigations of the vacuum state continued, eventually leading to Boyle's Law, the kinetic theory of gasses, development of chemistry, laws of thermodynamics, and the classical theory of the atom.

Although this development was initially in defense of a vacuum state, a space filled with substance to maintain matter was part of it as well.

The Kinetic Theory of Gases

Giovanni Batiste Beliani (1582-1666) debated with Galileo on how to explain vacuum effects. In the year 1620 the debate focused on the effect of air weight. It was known water could flow higher up from a vessel through a tube lying over a hill. However, if the top of the vessel was sealed, partial vacuum occurred at the top of the vessel from leaked water at the bottom, as to restrict the water from flowing through the tube. It was suggested by Galileo there are attractive forces between the water and the vessel, whereas Beliani believed outside air exerts pressure on the water attempting to come out the tube at its other end. Beliani was correct, as further investigation of the vacuum states led to such new inventions as the mercury barometer by Evangelista Torricelli (1600-1647) and the air pump by Otto von Gueriche (1602-1686).

Torricelli used the barometer to compare pressure of air at sea level to its pressure higher up in the mountains. He not only found a difference, he further discovered the pressure changes with a change in the weather. With mercury thirteen times denser than air, it is able to create a vacuum in a tube that varies according to present pressure of the atmosphere. Gueriche invented the air pump to produce more vacuum in order to measure more work capacity of outside air pressure. The outcome was him using the barometer in 1660 for forecasting the weather. He also published books, which Boyle read, defending the vacuum state proposed by Galileo and Newton and opposed by Aristotle and Descartes.

More experiments occurred in England, as Henry Power (1623-1670) and Richard Townely (1629-1707) examined air below atmospheric pressure in discovering the product of pressure and volume stayed constant. Robert Hooke (1625-1703) experimented with the air above atmospheric pressure to determine the same result. He and Robert Boyle (1627-1691) confirmed the nature of gas pressures more in general. Boyle then proposed, in 1662, the law that the product of volume V and pressure p of gas is constant at a fixed temperature T, as expressed by the equation pV = k. (Also, in France, this law was proposed in 1676 by Edme Mariotte (d. 1684).)

A constancy of volume-pressure relates to Newton's laws of motion in that a sphere with twice the radius of another has eight times more volume and four times more surface area, but it also has twice the average distance for a particle to reach the surface for it to collide half as often. The intensity of collisions in the larger sphere in relation to twice distance and four times more surface area is thus oneeighth as much pressure. Since the decrease in pressure is the same as the increase in volume, their product is the same for all spheres.

To understand the relation in terms of kinematics, consider pressure p as a force per area, as pounds per square inch, where weight is synonymous with the centripetal force of gravity, mv^2/r , and pressure as force per area is mv^2/r^3 , the same as per volume. As with the internal action, it is such that a particle moving twice r with same speed collides with the surface one-eighth as often, such that the product of one-eighth pressure with a volume eight times larger is constant. Indicative also is particular temperature T, since 2T particles are twice kinetic energy of twice speed particles.

What followed from Boyle's law is a relation of heat and temperature. Although the motion of atoms or molecules causes them, all that physicists essentially knew about them is heat is a quantity contained of the mass and the temperature is only a measure of how much a particular substance such as mercury expands in relation to heat absorbed, but Boyle's law enabled them to be understood in mechanical terms of relative motion and mass.

Daniel Bernoulli (1700-1782) initiated the kinetic theory of gases along with his study of hydrostatics. Bernoulli advocated a mechanical theory in analyzing the kinematics of molecules in view of particle collisions, despite a general regard at the time that such a process is too simple to resolve the more complex nature of reality.

Although the pressure-volume product was determined as constant for a particular temperature, it was still questionable as to whether the constant k is the same for different temperatures. Guillaume Amontons (1663-1705) foresaw the ideal gas law pV = nkT before it became established, as stating in a 1702 paper that a product of pressure p and volume V equals a product of temperature T for the same constant k, such that either pressure, volume or both increase with the increase in temperature. He further considered the zero temperature in relation to zero pressure, which anticipated an absolute temperature scale established about a century and a half later.

Another form of the law, as Charles' law, became the law of volumes whereby the volume of the gas container increases with temperature instead of an increase in pressure. The law was proposed by Jacques A. C. Charles (1746-1823) in 1787. It along with Archimedes' principle of buoyancy, led to the invention of the hot-air balloon. It was also established quantitatively by Joseph Louis Guy-Lussac (1779-1850), in an 1802 publication, whereby one-degree centigrade change in temperature corresponds to a change in volume of the same pressure occurring as one part in 273 parts of the gas volume.

An ideal gas law was hinted at by Amontons in 1702. It was explicitly stated, in 1834, by Paul Emile Claypeyon in relation to Boyle's and Charle's laws. August Karl Kronig (1822-1879) derived it in 1856 in accordance with the kinetic theory of gases, as Rudolf Clausius did as well in 1857. However, Johannes van der Waals (1837-1923) disclosed, in 1873, the law is not ideal because of electromagnetic effects influencing the results. That is to say, not all the kinetic energy of mass converts to heat. It can convert to some other form of energy, such as electromagnetism, as radiation of low energy light.

Another criticism of the kinetic theory of gases itself was that internal motion of matter would likely cause it to explode every which way. It would not be until the middle of the 19th century until a counter argument would come forth with Clausius explaining that collisions between minute particles great in number obstruct their mean free path of escape. Constant collisions keep reversing directions for the total distance moved to be longer than the direct outward distance itself. The kinetic theory of gases was thus not to be accepted until revived in a statistical form by Clausius, Maxwell, Boltzmann and others in a later part of the 19th century.

The Substance of Heat

Heat as molecular motion had many proponents, including Boyle and Hooke, but until the later part of the nineteenth century a theory of heat as a substance was more accepted. The criticism pertaining to an inner, violent molecular motion causing matter to explode in all directions was influential. For this reason and others, the idea heat is a particular substance absorbed and emitted by matter instead of only an internal movement of the internal components of matter remained popular. However, debate on the nature of mass and heat continued its advance.

Both Boyle and Newton proposed fire consists of material substance, as the residue caused to burn and produce heat. Boyle experimented to find substances

that did not decompose, which he defined to be an element. The concept of the element eventually led to the discovery of the atom.

Etienne Francois Geoffrey (1672-1731) advanced in 1718 the idea that a particular substance of a compound (such as carbon of carbon monoxide) is replaceable by another substance (as by hydrogen to convert the oxygen of carbon monoxide into water molecules). He then contrived a table of 16 columns in demonstrating an order of replacements of known substances. This effort evolved into a table of affinities wherefrom element A instead of element B combine with element C because elements A and C having more of an attraction for each other than do elements B and C.

The table of affinities was utilized by Joseph Black (1728-1799), Henry Cavendish (1731-1812) and Joseph Priestley (1731-1810) to discover nearly all elements of permanent gases. Their discoveries led to an explanation of a weight oddity from combustion and calcinations of dissimilar materials that result from a combination of different elements further resulting in various combinations of exchange.

Newton had considered an inert matter contained by elastic forces of some kind of æther, and he offered explanation of heat as a repulsive force decreasing in inverse proportion to distance between molecules. He argued light particles excite æther, which then conveys the excitement onto matter to produce heat along with other effect. He even supported the connection of æther with heat and light by experimentation on the bases a vacuum did not prevent a transfer of heat (as radiant heat requires no material medium to move through space).

Attraction and Repulsion

Newton was also influential to future theory for advocating a dualistic principle of both attractive and repulsive forces of nature: one being gravity and the other as heat. Stephen Hales (1677-1761) developed an idea of both attractive and repulsive forces consisting of two kinds of matter tending to become balanced in a state of equilibrium. His idea was to have a profound influence on Benjamin Franklin (1706-1790) and others with regard to two fluid theories of electricity and magnetism.

Theorists were generally inclined to relate all forces as substances of a particular kind determining the internal natures of matter. In the 1740s, for instance, Franklin proposed that electrical phenomenon is an elastic fluid of mutually repulsive particles. Matter is electrically neutral for containing the right amount of a particle fluid, repulsive if it contains an excessive amount, and attractive if deficient of it. Gowin Knight (1703–1772) also proposed a fluid for magnetism with the propagation of light as the vibrant motion set up in the fluid.

William Cullen (1710-1790) advanced the idea natural forces result as various modifications of æther. He proposed electricity, light, heat, gravity, magnetism and so forth emanate as various æther forms that are themselves gravitationally weightless and distinct from matter, thus allowing additional effects deviating from an

equilibrium state of gravity. Such a proposal was in line with the beliefs of Gassendi and the public in general.

Latent Heat

Cullen's student Joseph Black also considered heat, light and so forth as modifications of æther. In support of his consideration, he systematically studied combustion and calcination of different elements in pioneering the science of calorimetry.

Black's study pertained to temperature and heat. Newton had defined a quantity of heat as the amount of time taken to lower a substance to room temperature. Black took this definition to mean that heat can be measured as a time required for either dropping or raising its temperature to a certain degree. He measured a certain quantity of water according to the amount of time it takes to raise its temperature one degree. The temperature, however, did not change in such cases as involving change from water to ice, or vice versa, and from this find Black proposed the concept of latent heat.

Latent heat had previously been noticed by Daniel Gabriel Fahrenheit (1686-1736). He discovered water remained in the liquid state while cooling at the freezing point (as now 32 degrees Fahrenheit), but it congealed into a solid state above this temperature by means of shaking the container. This find that Black verified on his own in 1761 suggested heat can be stored as a latent form of energy without a rise in temperature. A mixture of ice and water, for instance, need not change in temperature with change in heat if a change in heat is relatively slight.

From Phlogiston to Caloric

In an effort to disprove a theory that phlogiston is a primary substance of fire, Antoine-Laurent Lavoisier (1743-1794) probably introduced caloric in naming the main substance of heat. George Ernst Stahl (1660-1724) had proposed phlogiston is a substance of violent motion producing flame and heat when entering into the air from the dispersing of matter. Plants absorb and then recycle it. There is truth to this idea if we identify phlogiston with carbon, which Stahl did, but he also claimed phlogiston is the only element existing not as a compound, but as a primary substance and a catalyst for all processes of combustion.

Stahl's generalization was open to dispute since some substances lose weight during calcinations whereas others gain weight. Thus, in some cases, phlogiston need be of positive weight; in other cases, it need be of negative weight.

A resolution to this quandary was the table of affinities first presented by Geoffrey; wherefore came many chemical discoveries. One of particular pertinence was the discovery of oxygen. Because of it, physicists no longer needed phlogiston to explain changed states of matter. Calx plus phlogiston producing metal, for instance, could more easily be explained as calx giving up oxygen to become metal. Conversely, metal absorbing oxygen becomes calx. Furthermore, experiments by Lavoisier indicated conversion of sulfur, phosphorus and arsenic into oxides results in a gain in their weight and the decrease in the weight of air.

In disclosing the inadequacy of phlogiston theory, Lavoisier held onto the general belief of Cullen and others that æther acts as a weightless elastic fluid that is not influenced by gravity, and it is primary responsible for such effects as light, heat, electric-static repulsion and so on. He followed up on experimental results of Cullen on cooling effects of such volatile liquids as alcohol by vaporization to discover, independently of Black and Fahrenheit, the temperature of ice did not increase while changing into the liquid state. Along with replacing phlogiston with caloric, he proposed an element such as oxygen has more affinity for absorbing caloric than does another, which could also allow ice to change to water while maintaining a slight portion of its coolness, as by a base material. The caloric thus seemed to provide more consistent explanation of how change in weight occurs of substance, but it was to be overcome with the development of atomic theory and the laws of thermodynamics.

From Caloric to Atomic Theory

It was evident substances combine in definite proportions, as a precise amount of oxygen combines with a precise amount of hydrogen. This find was named by Joseph Proust (1754-1826) the Law of Definite Proportions. It and a similar law led the way to the modern theory of the atom.

The similar law is the Law of Multiple Proportions proposed by John Dalton (1746-1844). According to it, the chemical elements consist of tiny particles called atoms. Atoms of a particular element are all of the same size, weight, mass, etc. that differ from those of other elements, but the different elements combine in ratio of whole numbers to form chemical compounds. Dalton proposed a numerical list of atomic weights of six known elements: hydrogen, oxygen, nitrogen, carbon, sulfur and phosphorous. Hydrogen, as the lightest of these elements, was assigned the number one.

Another similar law, proposed by Gay-Lussac, is the law of combining proportions. He and Alexander von Humboldt (1769-1859) discovered that two volumes of hydrogen combine with one volume of oxygen to become two volumes of vaporized water having the same temperature and pressure. Gay-Lussac further studied data collected by Humphrey Davy (1778-1829) with regard to volume ratios obtained by combining nitrogen with oxygen. He experimented to further discover a half gaseous volume of nitrous oxide is obtained by combining a volume of nitrogen and a half volume of oxygen of the same temperature and pressure.

Similar finds encouraged Gay-Lussac to conclude that gases combine in whole numbers ratios, as they also do according to weight. However, he did not explain why some whole number ratios differ from others. Why, for instance, does water vapor squeeze one volume of oxygen and two volumes of hydrogen into two volumes total? Amedeo Avogadro (1776-1856) explained the ratios of whole number combos according to a hypothesis a gas with the same volume, pressure and temperature of another contains the same number of molecules. As to why two volumes of water vapor is the result of two volumes of hydrogen gas and one volume of oxygen gas, he assumed molecules are formed from "solitary elementary molecules", or atoms. Since hydrogen gas contains two atoms for every molecule, two hydrogen atoms combine with an oxygen molecule, as a single atom, to form into water consisting of two different molecules. The number of molecules per volume thus stays the same; wherefrom, a specific number of molecules is the same. Heavier molecules merely add weight.

Avogadro's hypotenuse became known as Avogadro's Law. However, it was at odds with Dalton's atomic theory that assumed the compounds of elements are the result of like atoms repelling each other for allowing other kinds of atoms to occupy the space instead. Thus, Dalton and his followers were not to accept the concept that identical atoms combine in becoming a molecule. As it were, Avogadro's law failed acceptance until a fellow Italian, Stanislao Cannizzaro (1826-1920), pointed out in 1861 that the law could be used for a convenient table of atoms in simple ratios of whole numbers.

Rudolf Clausius (1822-1888) later helped promote the kinetic theory of gasses in explaining the vibrant motion of atoms is stable by being great in number, as each encounter between numerous atoms acts to slow the mean free path of escape by means of it constantly reversing directions, which is consistent with an interpretation of gravity by Newton according to vibrant cells or vortices of æther having a zero total momentum internally for each cell, but which implies that space is indeed filled with an enormous number of miniature cells as a medium of interaction.

The Fate of the Caloric

The role of caloric in developing theory was to replace the phlogiston of fire as an explanation of latent heat. Properties ascribed to caloric were it consists of weightless particles repelling each other, as flowing from hot to cold matter. Because caloric was also assumed to be conserved, an engineer named Nicolas Leonard Sadi Carnot (1796-1832) used the caloric theory for the derivation of theorems to explain how a steam engine is able to operate more efficiently.

Postulating conservation of heat in view of caloric, Carnot theorized it flows from hottest to coldest parts of the mechanism performing work, as caused by the viscous flow of caloric. Since caloric is conserved, the process compares to a waterwheel turning by the flow of water that forever recycles. Similarly, because systems of the same temperature are unable to exchange caloric to perform work, recycling of caloric is required. Adding more fuel, for instance, allows a hot steam engine to continually release caloric for it to perform work.

Carnot's condition for efficiency was correct for the most part, but the conservation aspect of caloric is inconsistent with conservation of energy in that energy merely converts from one form to another. Caloric was thus to become discarded in favor of energy conservation.

Neither physicists nor chemists had yet established the modern law of conservation of energy, but Count Rumford (1753-1814) had demonstrated an enormous quantity of heat results from boring cannon holes. Humphrey Davy (1778-1829) rubbed ice plates against each other to demonstrate heat is producible below freezing temperature even though no caloric should be available from it. Moreover, no appreciable amount of change in mass or of its weight occurred in either of these experiments. Rumford thus proposed vibrant motion causes heat instead, whereas Davy considered heat occurs as resulting from the absorbing of light. Davy also suggested a novel idea: that light combines with oxygen to become "*phosoxygen*", as the process whereby mass increases by absorbing light, which it does, in fact, in accordance with relativity theory.

Conserving Energy

The main difference between conservation of caloric and conservation of energy is caloric does not convert into other forms of energy whereas all energy does, as according to its modern concept. This modern concept was stated in 1841 by Julius Robert Mayer (1814-1878) as a force (then regarded as a varying form of energy) that merely changes from one form to another. It is thus neither created nor destroyed. He argued the loss of kinetic energy during inelastic collision between masses transforms into heat as an internal form of continuous motion.

James Prescott Joule (1818-1889) verified Mayer's argument in relation to friction. Heat that results from stopping motion by friction was common knowledge. Joule measured it quantitatively in relation to the magnitudes of heat from work used to overcome friction. It was also known that the flow of electricity through a highly resistant wire heats the wire. Joule established this effect quantitatively in 1841 in confirming current transforms into heat in mechanical units of work.

In 1847, Hermann von Helmholtz (1821-1894) addressed the scientific community in stating there is no such thing as a perpetual motion machine performing work without compensation for it in return. Explanation refers to an isolated system in that the internal energy of the system comprises, as stated in modern terms, a total amount of kinetic and potential energies of the molecules remaining constant until acted on by some external influence. Whenever interaction occurs, a change in internal energy is according to the quantity of heat absorbed as the amount of work performed on the system, as according to the equation

$\Delta E = H + W$

H denotes quantity of heat, W denotes the amount of work performed, and E proceeded by the Greek delta letter Δ denotes the change in energy.

Note that one form of energy is related in terms of its potential. A ball at some height, for instance, has a gravitational potential, as weight that can be converted to kinetic energy. If it falls from the table, then kinetic energy further encounters friction of the ball falling through air and colliding with the floor, converting further into heat energy. Potential energy thus links to force regarding storage of energy. Moreover, there is also entropy as stored energy to consider as lost for useful work unless allowed by some change in its surrounding environment, as by another energy source.

Entropy

Clausius, in 1850, reformulated the theorem put forth by Carnot for it to comply with conservation of energy. It was also reformulated in 1851 by William Thompson (1824-1907), renamed Lord Kelvin. In result, a concept of entropy was established as the second law of thermodynamics.

In view of energy conservation in contrast to the caloric, adding fuel is to sustain a difference in temperature for doing work, but there are various forms of energy, and there is further distinction between useful energy and non-useful energy. Useful energy is that which can be used to create change; non-useful energy is energy in a state of equilibrium not changing unless by outside influence. For instance, two bricks in thermal equilibrium, as being of the same temperature, are incapable of doing thermal work by way of an exchange of heat from one brick to the other, but if one brick encounters a cooler environment, then potential heat energies become active.

It is therefore possible to have a certain amount of energy in the form of heat at absolute temperature, say T_1 . In theory, we can only harness all of the energy of a system if its temperature is reduced to absolute zero, as by a remarkably efficient machine of absolute zero temperature that is capable of harnessing it. Generally, however, systems are somewhere between absolute zero whereby the energy can only be harnessed by lowering it from T_1 to T_2 according to the relation

$$1 - \frac{T_2}{T_1}$$

Applicable to this condition is an absolute temperature scale, which Kelvin introduced after Joule suggested in a letter to Kelvin that it was possible to measure the difference from absolute zero. All heat energy is thus available if $T_2 = 0$; none is available if $T_2 = T_1$. (The absolute temperature scale allows for zero temperature; the fraction T_2/T_1 is the same for any scale.)

There are such other forms of latent or stored energy as that of atomic energy. Clausius, who had already considered the thermal aspect of it, went a step further to generalize the process in the form of a thermodynamic law he named entropy. Its general conditions are: Entropy is the measure of the energy of an isolated system not able to do work; it does not decrease other than by it increasing the entropy of another system.

For an example of increasing entropy consider an enclosure having no outside influence except for gravity. Inside it a string holds a rubber ball to prevent it from falling. Eventually the string breaks to allow the ball to fall in converting potential energy to kinetic energy. Assuming the ball and floor to be nearly elastic, two forms of energy (potential and kinetic) alternate as the ball bounces, but the ball also encounters the friction, or viscosity, of air while in motion, and the process tends to be inelastic. Even if there were no air, it is still inelastic because of heat generated by the ball's impact with the floor. With the kinetic energy converting to heat energy, the ball eventually comes to rest on the floor, as given for increasing the relative speed of the floor molecules passing on added motion to the outside matter they heat up in turn. Being in a state of equilibrium that is irreversible except for outside influence, entropy is thus increased to a maximum until another part of the universe is able to intervene. If the universe is this isolated system, then it is the result of a heat death, as a state of thermal equilibrium, unless a limiting factor of entropy somehow exists. Gravity, for instance, could be the means of a recycling process by creating weight, pressure and heat.

Heat quantity Q in ratio to absolute temperatures is thus a determining factor of entropy of internal bodies of an isolated system, as the difference in temperatures of the bodies determines their amount of heat available for useful work. A body at temperature T_1 thus surrenders its heat Q to another body at temperature T_2 for a change in entropy, ΔS , of the two bodies to be

$$\Delta S = \frac{Q_2}{T_2} + \frac{Q_1}{T_1}$$

If a system performs work, then the amount of heat Q_1 lost at temperature T_1 will generally differ from an amount of heat Q_2 gained at temperature T_2 , but if the positive amount given up is the same as the negative amount received, such that $\Delta S = 0$, then entropy is conserved as an adiabatic process.

FROM WAVE THEORY TO RELATIVITY

Theories of nature evolved from opposing views pertaining to how space is filled. By one, the internal nature of matter comprises indestructible atoms moving by way of a partial vacuum. By another, a primary substance fills all space as a plenum.

A belief in the plenum was popular from the time of Aristotle until the development of fundamental laws of motion. After the establishment of the Copernicus heliocentric view of the solar system, and Galileo and Newton establishing their mechanics, it became more acceptable Earth moves freely as though the space before it is empty except for it only being partially filled with tiny indestructible atoms. Nonetheless, such philosophers as Descartes proposed, to the contrary, atoms are tiny vortices of swirling æther to only appear as though they move as particles through empty space.

Nowadays the majority of physicists consider æther as non-existent, or invalid, since it is invisible in theory, and since physics confines itself to only what is observable, but it nonetheless has a long history in the development of theory, which wave action is an integral part of, as illustrated by the chain reaction below.



The illustration above depicts how momentum continues, as by either a medium or empty space before it. Through the medium the momentum is transferred to and from each iron ball by means of impulse. There is thus a momentum of impulse moving from one end of the row to the other when the former is struck by another ball. In this case, the momentum of the ball continues to reemerge as relative motion of another particle.

The particle description is simpler, but the wave action is also helpful for understanding the true complexity of nature. Newton, for instance, was unable to explain the cause of gravity by way of contiguous action. He thus described it according to an "action at a distance principle". Although other attempts have been offered to explain the direct cause of gravity, none have been successful for centuries to come. Wave theory, on the other hand, can be more extensive in its approach. Waves, for instance, can superimpose to negate their relative effects, as to explain gravitational action at a distance as wave action through an undetectable medium.

Wave action, however, is extremely complex with regard to the many ways it can occur, depending on the nature of the wave producing medium. Sound waves, for instance, propagate longitudinal action whereas transverse action is more typical of the wave property of light. Most waves are periodic in nature, such that periodic motion of a particle in general can be described in accordance with a wave equation, but surface waves of the ocean vary in their periodicity with regard to a change in water depth.

A wave action is insightful for anyone able to grasp its complexity, but such complexity is here avoided in favor of restricting it to a history of light.

Light and Wave Theory

Waves are noticeable events, such as surface ripples on a pond that are created by some sort of disturbance. In relating their wave action, Aristotle proposed light occurs from a wave-like disturbance of air. However, as far as is known, there was no constructive wave theory of light until 1678 when Christian Huygens (1629-1695) proposed his.

Huygens offered a principle wave-envelopes are created anew at every point in space a wave impinges on. Their creation spreads in all directions, but supposedly (as without any known explanation by Huygens) waves that are created back towards the center where the original disturbance occurred are obliterated by overlapping of continual creation, as to restore a state of equilibrium. Only the outer envelopes continue to spread from the center.

The theory has been criticized for its lack of explaining the obliteration effect, but Huygens likely had in mind that the wave action complies with the laws of motion. His understanding of them is evident with regard to his publication of centripetal force. He could have perceived the obliteration of wave action as mutual cancellation of momenta, whereas advancing waves merely carry momenta forward. In analogy to the impulse action of the iron balls in a row, elastic action of space can allow an excess amount of momentum to move forward while the obliteration of waves is a recycling process for maintaining a state of equilibrium for creation of more waves.

Huygens was able to explain double refraction in accordance with his theory. However, reflection and refraction were more simply explained in a manner consistent with a particle theory of light.

The law of refraction was accurately explained in a manuscript by Ibn Sahl (940-1000) of Bagdad as early as 984. In 1021, it became promoted in a treatise on optics by Alhazen (965-1014), who viewed light as consisting of rays of particles. It

was first formulated by Willebrord Snellius (1560-1626) in Europe to become known as Snell's Law.

Descartes later introduced the sine function for the ratio of angles.

The direction of a stick submerging into water appears to change, but the change is an optical illusion. Instead of the stick changing direction, the light from it has changed directions twice by entering into and leaving from a denser medium, as refracted. Whereas reflection is simpler in that it equals the angle of incidence, refraction entails change in speed of waves entering into a different medium from which the ratio of angles of incidence Θ_1 and refraction Θ_2 equate with the ratio of speeds v_1 and v_2 , and with the inverse ratio of refractive indexes n_1 and n_2 of the two mediums:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\mathbf{v}_1}{\mathbf{v}_2} = \frac{\mathbf{n}_2}{\mathbf{n}_1}$$

Huygens used his wave theory to explain double refraction of light, as discovered by Rasmus Bartholin (1625-1698) in 1669 to occur in the calcite crystal called Iceland spur. In double refraction, the light rays split into two directions, one ordinary and the other extraordinary. The ordinary direction complies with the law of refraction, whereas the extraordinary direction is a non-compliance of the law. An explanation of this non-compliance is given by Huygens' theory, as ripples of the medium spread waves in all directions. Even more complete explanation was to be given with the development of wave theory to include properties of interference and transverse vibrations.

The corpuscular theory of light remained in favor until Thomas Young (1773-1829) proposed a principle of interference. Leonardo da Vinci (1452-1529) had observed water waves are able to cross paths without obstructing each other's movements. Young applied this wave effect to light waves, and he included such additional effects of the interference of the waves crossing over each other paths. He surmised waves superimposing combine to either increase or decrease their effect depending on the degree they are either in or out of phase. If waves overlap in phase, then they combine their effect. If waves overlap in opposite phase, then their effect is canceled for a dark spot to appear. For conservation of energy, it is change in effect rather than the effect itself, as opposite but mutual changes in energy effects.

Young explained light and dark fringes in diffraction patterns of waves according to his principle of interference. Francesco Maria Grimaldi (1618-1663) had discovered sunlight spreads abnormally when it passes through a small hole. Young drilled two holes for an experiment from which he found light and dark fringes occurred when they otherwise would not if only one hole was drilled. The fringe pattern is explainable as the diffraction patterns of two rays of light overlapping in and out of phase. Where they overlap in phase, light amplitudes combine to become one; if they overlap in opposite phase, no light appears. (The amplitude in this classical sense simply means the height of the wave from crest to trough.)

Some experiments produce no interference, which is now explained as light emitted from the source as relatively incoherent. Since light waves are extremely rapid, and since trains of them are relatively short, it is possible to detect two or more rays superimposing only in particular circumstances. It occurs in Young's experiment by a drilling of holes close together. Another method is to split a light ray into two components, as for partial reflection. The split parts move through different paths of slightly different distances for them not to rejoin in their original state; they then superimpose instead into a slightly different state than their original one.

Transverse Waves

Young was able to explain nearly all light effects except polarization, a condition whereby particular direction up-and-down or sideways-back-and-forth is perpendicular to the direction that the waves move forward. To the contrary, longitudinal sound waves are three dimensional compressions that move forward instead of as a plane moving forward, as thus not polarized. In 1808, Etienne Louis Malus (1775-1812) discovered that either reflection or refraction can produce polarization. Young had not explained it because his medium for light waves compared to the propagation of sound through air as longitudinal waves of rarefaction and condensation of the air medium.

Young did suggest in 1817 that the waves needed to contain transverse components for an explanation of the polarization effects. Augustin Fresnel (1788-1827), who was unaware of this suggestion, formulated his theory of optics in 1818 that did explain polarization as transverse waves. His theory includes Young's principle of interference and also the principle set forth by Huygens of continuous waves spreading from all the points of contact with space. However, the transverse wave led to another enigma inasmuch as the transverse wave does not normally occur in a three-dimensional solid state. A rigid medium for three-dimensional space seemed to be too much of an obstacle to explain how planets and other objects move as freely through it as they appear to do.

The Elastic Medium

In investigating properties of the elastic medium, Claude Louis Navier (1785-1836) assumed objects are of extremely minute and compact particles whereby attractive and repulsive forces counteract in maintaining a state of equilibrium. The restoring force is analogous to a liquid reacting according to the motion of the particles. The solid state is conditional to the distance of separation between particles.

Navier's theory was elaborated on by Augustive-Louis Cauchy (1789-1857) with a law of elasticity that Robert Hook (1635-1703) had proposed. According to it, the stretching of an elastic body is proportional to the force applied for the stretching.

Cauchy interpreted Navier's theory as setting up a condition of strain. Physicists name the mathematical formulation of this condition a tensor in relation to a more complex vector quantity in applying to variable forces of higher order. A vector refers to a quantity having both particular magnitude and direction. A current, for instance, displaces the path of a boat crossing a river at a given speed. A vector is speed of the boat moving in one direction and the current force moving in another direction. The tensor could include an increase in the speed of the current, say, due to it nearing a waterfall.

Cauchy's results were mathematically consistent with those of Navier's homogeneous media, but more than one elastic constant of proportionality is needed for isotropic media. Whereas a homogeneous medium is the same everywhere, an isotropic one is the same only in directions, as it can vary in distance and other aspects. The media is necessarily isotropic in the case of Cauchy's results in allowing more than one kind of wave (as transverse and longitudinal) to propagate through it.

The equations of elastic solids were incompatible with optics insofar as they allowed for a longitudinal vibration as well as a transverse one. Cauchy overcame this incompatibility by considering a labile æther, as also capable of changing to a negative compressibility. This negative compression allows the æther to react differently to various kinds of waves, and even to allow the longitudinal velocity to be zero, as for standing waves.

George Green (1793-1841) investigated Cauchy's results, and he found them to be inconsistent with conservation of kinetic energy. They were then criticized by Simeon Denis Poisson (1781-1840) and Franz Ernst Neumann (1798-1893) as inconsistent with a wave theory developed more completely by Green.

Electromagnetic Rotation

As to how the planets and other objects move so easily through æther of such solidity, ideas came forth. Gabriel Stokes (1819-1903) suggested the effect is relative. The æther relates to the slow-moving planets as a rarefied fluid, or jelly, but to the extremely rapid vibrations of light as a solid. James Mac Cullagh (1809-1847) proposed ethereal vortices or atoms do not resist a displacement resulting in distortion of the medium; they change instead in their state of rotation. This process is of a transverse nature allowing atoms to move freely through æther with rotations subject to luminous effect.

With these two ideas combined, matter moves through ather without resistance, as if sinking into jelly, whereas light occurs as the changes in the rotational states of atomic-like vortices. As rotation allows movement in the plenum, there emerges an infinitely complex variation of rotations to allow unlimited effects of light, electricity and magnetism from different types of wave action.

Such speculative ideas were followed by empirical discoveries. In 1820, Han Christian Oersted (1777-1851) tested the effects of a magnetic needle near an electric current induced in a wire from a battery. He discovered the wire deflected the

needle, as to reveal a connection between electricity and magnetism. Francois Arago (1786-1853) then discovered an electric current magnetizes iron. Andre-Marie Ampere (1775-1836) then demonstrated that electric currents affect each other similar to the attraction and repulsion of magnetic poles. Electric currents repel each other when flowing in opposite directions; they attract each other when flowing in the same direction: which is explained by a right-hand rule as to why opposite poles align close together.

In 1831, Michael Faraday (1791-1867) discovered change in a magnetic field induces an electric current in a wire. It is thus only necessary to apply a force, such as wind or whatever to moves poles of a magnet for producing an electric current in a coiled wire that further produces additional magnetic effect. As an electric current produces an electromagnet, the electromagnet, in turn, produces more current. Alternating poles of the electromagnet near to the wire thus results as electromagnetic induction, as to how a generator is able to transform mechanical work into electricity.

A law to equate an electrical current to a magnetic field was developed by Jean-Baptiste Biot (1774-1862) and Felix Savart (1791-1841), and also by Pierre Simon Laplace (1749-1827). A significant part of the law is a constant of proportionality c, as equating a unit of electric charge e per time t passing through a unit length d of a section of a wire in proportion to the magnetic pole strength p, as cp/e = d/t. The pole strength p of the magnetic field is of the same dimensions as a unit of charge e of an electric field. They cancel each other out in the equation for c to be identified as a velocity: v = d/t.

Wilhelm Eduard Weber (1804-1891) and Rudolph Kohlrausch (1809-1858) ascertained in 1836 a value of c being the same as light speed, namely about $3 \ge 10^{10}$ centimeters per second.

The constant of proportionality c having the dimensions of a velocity was significant for the formulation of electromagnetic theory. Ampere had believed magnets are particular parts of electromagnets induced by electric currents within molecules of matter instead of the wires. However, Faraday believed magnetic currents, or "lines of force" in his way of thinking, exist in virtually quasi empty space whereby changes occurring in electromagnetic fields take time, whereby the propagation of their effect is the propagation of light.

Faraday did not formulate a mathematical theory. His ideas along with others were included in a theory of electromagnetism formulated by James Clerk Maxwell (1831-1879). According to it, material wires are not needed to conduct electricity, as it is able to propagate in the continuum of space as provided by the presence of an electromagnetic field alone. A displacement of electric current simply produces an electric field that induces a magnetic field that, in turn, induces another electric field, etc. An open field creation thus progresses at light speed as the electromagnetic spectrum.

Included in Maxwell's formulation of electromagnetism are Coulomb's Law and the Biot-Savart Law. An inverse-square law for electrical force had been proposed by Joseph Priestley (1733-1804) and others. It was published first by Charles Augustin de Coulomb (1736-1806). Similarly, but related to a current of charge, is the Biot-Savart Law proposed by Felix Savart (1791-1862) and Jean Baptiste Biot (1774-1862) whereby the magnetic intensity between any two points along two different parallel wires is proportional to the distance squared between them and the amount of current flow and their speeds. If the currents flow in the same direction, they attract; if they flow in opposite directions, they are repelled.

The two laws are united by Maxwell's theory of electromagnetism by a relationship of magnetic permissibility and electric permittivity according to light speed. Constants of magnetic permissibility and electrical permittivity are denoted as μ and ε , respectively. Along with light speed c they are c₀, μ_0 and ε_0 in vacuum space, absence of mass, equating in the manner

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$
$$c_0^2 \mu_0 \varepsilon_0 = 1$$

It is with the presence of mass whereby the above constants have empirical effect. For either μ or ε or both greater than μ_o or ε_o light speed c needs to be less than c_o in order to maintain unity.

Advancing Theory

A verification of Maxwell's theory of electromagnetism came in 1888 when Henrock Rudolf Hertz (1857-1894) produced electromagnetic waves in showing that they interfere with themselves, and he even measured their wavelengths as fringes produced on a screen.

The next task became to determine the state of the æther in relation to matter. For instance, if light speed c is constant through the æther, it is also consequential whether æther itself is in a state of motion that can or cannot influence the presence of matter.

Three possibilities were considered: 1) no æther is carried by matter; 2) matter partly carries æther in its path; and 3) matter carries all æther within the confinement of its path. Hertz adopted the third hypothesis, which had already been presented in 1845 by Stokes, but it was shown as inconsistent with the experimental results. More consistent with experiment was a partial drag hypothesis, which had been proposed by Fresnel. A theory even more consistent with experimental results in general was one offered by Hendrick Antoon Lorentz (1853-1928), which assumes æther, as a state independent of matter, is a particular state of absolute rest. Although the assumption was not verified, further explanation led to the special relativity principle, which can explain all experimental results by assuming the properties of matter are altered to move through an æther as invisible. Clocks, for instance, become slower and length of matter in the direction of motion becomes shorter.



A particular experiment of significance at the time, as illustrated above, was one suggested by Francis Arago (1786-1856) indicating matter drags the æther. As light rays are split by a partially coated silver mirror, other mirrors direct the split parts for them to pass in opposite directions through a glass tube of running water. Rays moving in the same direction the water moves arrive back sooner than the rays moving in an opposite direction. However, the increased speed of some light and the decreased speed of other light are only equal to a fraction of the water's speed, as in agreement with Fresnel's partial drag hypothesis.

This experiment supported Fresnel's partial drag hypothesis, but it was not able to explain another experiment by Edward Williams Morley (1838-1923) and Albert Abraham Michelson (1852-1931) in 1881, as not including the running water. If æther is either dragged or partially dragged by matter, then the slight shift in fringes of the wave pattern on a screen by this similar experiment should indicate a change in direction of Earth as it rotates on its axis and revolves around the sun. To the contrary, however, no appreciable change was observed to occur.

Explaining Experiment

The result of one experiment was consistent with Fresnel's partial drag hypothesis. Another experiment favored Stokes' complete drag hypothesis. A more complete theory was needed in order for it to explain experimental results in a more consistent manner. A near candidate at the time for such a theory was Lorentz's electron theory.

Lorentz had theorized electromagnetic fields contain minute particles that bond together by their mutual attraction from having opposite charge. They are thus a mutual state of vibrant equilibrium. Such a state affects light propagating through it. Because an electromagnetic field produces light, the state of the field itself is also changed. As the electromagnetic waves of light interfere with the equilibrium state of charged particles, the field reacts to a changed vibrant state of charged particles. The resulting effect is there is an increase in inertia of matter as it moves through the æther, as to vibrate at a slower rate and as having more external momentum.

The vibrations of matter also replace the æther as a light medium, as is typical of water or glass changing the direction and speed of light, but light propagating through material mediums differ from that of æther. A material medium varies in its relation to variable light whereas the æther is the same for all light. The different size light waves move at different rates depending on the relative density of the material medium. The light that moves in the same direction as the water is therefore faster than if moving contrary to it. Matter being a medium of light propagation instead of it dragging æther as the medium explains the result of the Arago experiment. However, it failed to explain the Michelson-Morley null results.

To explain the null results of experiment, Lorentz assumed that matter moving through æther is squashed in the direction of relative motion. The contracted length of the apparatus in the relative direction of motion results in light moving the same distance it would if at rest in the æther.

Although contraction of length is sufficient by itself to explain the null results of the Michelson-Morley experiment, Lorentz went a step further in defining a "local time" in assuming clocks are similarly affected by moving through æther. Hence, a time of propagation is the same as if the apparatus is at absolute rest with the æther, suggesting the state of absolute rest in the æther is not ascertainable by means of the Michelson-Morley experiment.

Jules Henri Poincare (1854-1912) urged Lorentz in 1900 to generalize his theory to comply with the principle of relativity wherefore all motion of mass is merely relative, being no observable state of absolute rest. Lorentz did provide transformation equations to this effect, but he maintained there might still be possible means of detecting a state of absolute rest. Meanwhile, Einstein independently derived the same Lorentz transformations in a manner that is free of all preconceived notions pertaining to the æther.

Lorentz and Einstein's Explanations

Woldemar Voight (1850-1919) had derived these same transformation equations in 1887 to describe a rigid medium of light as a non-compressible fluid. Voight considered the theories of Cauchy, Neumann and others in view of a Doppler effect propounded by Christian Doppler (1803-1853). By it, sound waves or light waves are either stretched or contracted depending on the relative direction of motion between the observer and the source of emission. However, neither Lorentz nor Voight applied the transformation equations more generally to the laws of mechanics. Einstein did as such. As a consequence of it and the development of quantum physics, the æther as a methodological part of scientific theory became discarded. In reformulating the principle of relative motion, Einstein emphasized the æther is unnecessary for formulating theory. He demonstrated this claim in 1905 with his formulation of special relativity theory for a unification of electrodynamics and mechanics, as it only postulated the constant speed of light as an empirical fact of nature with no reference to the æther.

Einstein did suggest the æther could be used for understanding theory, as just not being needed for formulation, but other physicists did not agree it should be retained. After physicists discovered both light and matter have dualistic particlelike and wave-like properties, leading physicists at the time, such as Bohr, Born and Heisenberg reinterpreted so-called wave equations as probability equations instead, as for determining probable location, time, energy and momentum for a particle effect to occur.

Leading into the twentieth century was these two revolutionary ideas, a relativistic and quantum natures of light and mass. They did not conflict for the most part with each other. For instance, a photoelectric effect explained by Einstein as light particles to be named photons became part of quantum theory. A similar effect to the photoelectric one is the Compton Effect that Compton formulated as quantum effects consistent with relativity theory. A discovery, in 1924 by Louis de Broglie, indicated both matter and light can be described as either duel wave-like or particle-like, and he theorized them in accordance with both quantum and relativity theories.

Initially the quantum could be formulated along with relativity theory, but it also developed as a contrary means of explaining nature. Whereas the relativity theory is deterministic with regard to cause and effect, a quantum mechanics became interpreted as indeterminate according to probability.

This condition of probability pertains to wave effects of nature. Erwin Schrodinger provided a quantum wave equation for de Broglie theory to be formulated as a wave mechanics whereby it is assumed an electron consists of clouds of standing waves producing a quantum-particle effect. However, subsequent development led to the Heisenberg principle of uncertainty by which the world on the atomic scale became considered indeterminate with regard to knowing cause and effect.

In 1927, top physicists of the time convened in Copenhagen, Germany in concluding a yearlong debate ending with the Copenhagen Doctrine. As it were, Schrodinger's wave equations had been shown to be successful for predicting events, but Born and Heisenberg argued for an impossibility of determining both exact position and exact momentum of a particle because the means of determining them (as by light) influences the outcome. Even though the Schrodinger's wave equations predicted outcomes, they were to be reinterpreted as probability waves.

Along with indeterminism was the demise of the æther as the means of explaining nature. Since the primary aim of science is to describe nature in accordance with observation, and because experiment indicates the æther is invisible if it does exist, it is thus non-existent as far as science is concerned. However, subsequent development has been shown to be illusive. A virtual field of virtual particles is needed to more accurately predict results.

A double standard also seems to exist with regard to explanation, as no explanation is required to explain how virtual particles can cause attraction, but tired light theory is dismissed for not explaining how distant stars move through the medium of space, lose energy and are still as visible as they are. Explanation of the latter is to be offered later on in this book.

SIMPLE SPACETIME RELATIVITY

Einstein formulated special relativity (SRT) according to two postulates: 1) the speed of light is constant in a vacuum relative to the observer regardless of what the velocity of the observer is relative to any other; 2) physics laws are the same in all inertial reference frames. The second postulate is known as a principle of covariance. Light speed is covariant, but it is unique as well for its speed in vacuum space being the same in contrast to a variable speed of matter.

How these postulates modify Galilean relativity according to Newton's absolute space and absolute time according to constant light speed and the relativity of spacetime is explained here in accordance with the null result of the Michelson-Morley experiment. It continues with derivations of Lorentz Transformations and so forth explained in view of modification of Galilean relativity and how observation of absolute rest is nullified.

The Michelson-Morley Experiment

The Michelson-Morley experiment was an attempted determination of light speed in relation to æther. If the æther represents absolute rest, and if light waves propagate at the same speed through it, then light speed should vary with Earth's motion varying through æther in various orbital directions around the sun. However, no significant variation of light speed was found by this experiment, or of any other experiment. Replacing absolutes of space and time with relative spacetime essentially explains the null results.

According to the nature of this experiment, light speed is measured as a tothere-and-back event. A silver-coated lens is positioned at a 45-degree angle to the incident light to split it into separate rays. One ray continues on through the lens in the same direction while the other ray reflects at a right angle. Other lenses reflect the rays back to the silver coated lens for them to pass through it and superimpose onto a screen.

What physicists expected from this arrangement is the total distance of each split ray differs because of perpendicular lengths of the apparatus not being precisely equal. What actually appears on the screen is an interference pattern of light and dark fringes (as are predicted of superimposed waves). An appreciable amount of change to verify the change in the speed of light is what was not observed.

To explain the null results, Lorentz assumed length s_x of the apparatus contracts in the direction of relative motion by the factor

$$\alpha = \sqrt{1 - \beta^2}$$

 β represents v/c as the direction and speed of matter in ratio to light speed. Perpendicular lengths s_x and s_y are considered equal:

$$s_x = s_y = s$$

Each arm of the apparatus compares as equal to a proper length denoted as s such that, if the apparatus is theoretically at absolute rest, then respective times for light to move respective distances there-and-back are according to the equations

$$2t_x = t_{x1} + t_{x2} = \frac{s}{c} + \frac{s}{c} = \frac{2s}{c}$$
$$2t_y = t_{y1} + t_{y2} = \frac{s}{c} + \frac{s}{c} = \frac{2s}{c}$$

In these equations, t_{x1} and t_{y1} are the respective times to there and t_{x2} and t_{y2} are the respective times back from there.

What if the apparatus moves at velocity v in the x direction relative to absolute rest?

According to Galilean relativity and the premise light speed is invariant relative to the æther, the times for light to move respective distances s_x and s_y vary. The differences in light speed and the apparatus speed are thus c - v in the direction of motion and c + v in the opposite direction of motion. A total time of the to-andfrom propagation along the x-axis should be

$$2t'_{x} = t'_{x1} + t'_{x2} = \frac{s}{c-v} + \frac{s}{c+v}$$

$$=\frac{s(c+v)}{(c-v)(c+v)} + \frac{s(c-v)}{(c+v)(c-v)} = \frac{sc+sv+sc-sv}{(c-v)(c+v)}$$

$$=\frac{2sc}{c^2-v^2} = \frac{2sc}{c^2\left(1-\frac{v^2}{c^2}\right)} = \frac{2s}{c(1-\beta^2)} = \frac{2s}{c\alpha^2}$$
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This time compares to $2t_x = 2s/c$, as if the apparatus were at absolute rest. If instead the apparatus contracts in the direction of relative motion by the factor α , then the difference from that of absolute rest is only by the factor $1/\alpha$.

Similarly, the total time it takes light to propagate in the perpendicular direction of motion, as during time 2t'_y, can be determined. The actual path of light is according to two directions: 1) the direction along the arm of the apparatus perpendicular to the direction of motion; and 2) the direction of motion in keeping pace with the apparatus. The actual distance is along the hypotenuse of a right triangle with respect to distance moved of those two other perpendicular directions. Hence

$$(ct'_{y})^{2} = s^{2} + (vt'_{y})^{2}$$
$$s^{2} = (ct'_{y})^{2} - (vt'_{y})^{2}$$
$$s^{2} = (t'_{y})^{2}(c^{2} - v^{2})$$
$$(t'_{y})^{2} = \frac{s^{2}}{c^{2} - v^{2}} = \frac{s^{2}}{c^{2}(1 - \beta^{2})}$$
$$t'_{y} = \frac{s}{c\alpha}$$

The time light moves to-and-from in the perpendicular directions is

$$2t'_y = \frac{2s}{c\alpha}$$

It is the same with respect to the direction of motion if the apparatus arms in the direction of motion contract by the factor α .

Because light moves at the same time, speed and distance along both arms of the apparatus, there are no differences observed of a shift in the pattern of fringes. Although this explains the null result of experiment, it only requires the apparatus to contract in the direction of relative motion; it does not require clocks to retard as well. However, it is just another step to determine them as relatively retarded according to relative motion through the æther, as to complete the analysis in view that absolute rest is invisibly non-discernible.

If clocks are slower by the factor $1/\alpha$, durations of events are shorter by the factor α . Hence

$$2t'_x \alpha = 2t'_y \alpha = \frac{2s}{c\alpha} \alpha = \frac{2s}{c} = 2t_x = 2t_y$$

Absolute rest is thus invisible with regard to there-and-back light speed.

This mathematical result constitutes a general explanation of constant light speed. What follows from it are more intricate explanations in relating such principles as covariance and simultaneity. With regard to the relativity of simultaneity, it is not possible to measure light speed, in view of absolute rest, by timing signals from one place to another by a clock that determines time of an emission as synchronous with a clock that determines its time of reception. A synchronization of clocks by comparing signals invalidates the measure of light speed from signals emitted by the clocks. To confirm two clocks are synchronous by direct means requires the transportation of one clock to the location of the other, as to involve relative motion that retards clocks. Since exact speed of the clock relative to absolute rest is unknown, the amount of retardation of the clock in motion is unknown as well. To be sure, mathematical analysis is to verify the relativity of constant light speed, as covariant, is maintained.

Covariance

Covariance means the laws of physics apply the same to all systems. In other words, the perceptions of observers **A** and **B** are the same with regard to **A** or **B** being relatively at rest and the other moving at speed v. Because motion affects lengths and clocks, it is not obvious the speeds of different observers are perceived the same relative to each other, but relative motion is easily verified mathematically as covariant.

Consider Observer **A** at absolute rest sees Observer **B** approaching at velocity v the distance from x_1 to x_0 during time $t_1 - t_0$. The event includes a time it takes light to move a distance between positions x_1 and $x_0 = 0$. The time t_d of Observer **A** seeing Observer **B** moving the distance $x_2 - x_1$ is

$$t_a = \frac{x_1 - x_0}{v} - \frac{x_1 - x_0}{c}$$

Since the clock of Observer **B** is slower by the factor $1/\alpha$, since Observer **B** approaches the oncoming light at velocity v, and since coordinate lengths of Observer **B** are relatively shorter by the factor α , the duration of Observer **B** seeing Observer **A** move along length $(x_1 - x_0)\alpha$, as according to observer **B**'s clock, is

$$t_{b} = \frac{(x_{1} - x_{o})\alpha^{2}}{v} - \frac{(x_{1} - x_{o})\alpha^{2}}{c + v}$$

The task is to show $t_a = t_b$:

$$\frac{x_1 - x_0}{v} - \frac{x_1 - x_0}{c} = \frac{(x_1 - x_0)\alpha^2}{v} - \frac{(x_1 - x_0)\alpha^2}{c + v}$$
$$\frac{1}{v} - \frac{1}{c} = \frac{\alpha^2}{v} - \frac{\alpha^2}{c + v}$$
$$c(c + v) - v(c + v) = c\alpha^2(c + v) - vc\alpha^2$$
$$c^2 + vc - vc - v^2 = c^2\alpha^2 + vc\alpha^2 - vc\alpha^2$$
$$c^2 - v^2 = c^2\alpha^2$$
$$c^2 \left(1 - \frac{v^2}{c^2}\right) = c^2\alpha^2$$

Since $\alpha^2 = (1 - v^2/c^2)$, the equality $t_a = t_b$ is confirmed. Observers **A** and **B** thus perceive relative motion the same if either A or B is at absolute rest.

Simultaneity

Even though Einstein postulated constant light speed as fact without explanation, his principle of simultaneity helps explain it. Because Observer **B**'s clock in relative motion is slow, it is not simultaneous with the clock of Observer **A** relatively at rest. However, by the principle of covariance **A** and **B** perceive events the same, such that **B** is also regarded as relatively at rest. Simultaneity of events is thus the same for both observers.

Consider **B** moves away from **A** at velocity v to a distance x after time t according to **A**'s clock. The task is to confirm **B**'s perception of time is the same as **A**'s. At distance x, it takes time x/c for light from **B** to move to **A**. The time for **A** to see **B** move the distance x is thus x/v + x/c. Because the coordinate lengths of **B** contract in the direction of motion by the factor α , and since duration of events are also shorter by the factor α , as timed by **B**'s slower clock, and because the difference in **B**'s velocity from that of light is c - v in relation to **A** being relatively at rest, an equality of **A** and **B**'s timing of the event is

$$\frac{t'}{t} = \frac{\frac{x\alpha^2}{v} + \frac{x\alpha^2}{c - v}}{\frac{x}{v} + \frac{x}{c}} = \frac{\frac{\alpha^2}{v} + \frac{\alpha^2}{c - v}}{\frac{1}{v} + \frac{1}{c}} = \frac{\frac{\alpha^2(c - v) + \alpha^2 v}{v(c - v)}}{\frac{c + v}{vc}} =$$

$$\frac{\alpha^2 v c (c - v) + \alpha^2 v^2 c}{v (c - v) (c + v)} = \frac{\alpha^2 c (c - v) + \alpha^2 v c}{c^2 - v^2}$$
$$= \frac{\alpha^2 c (c - v) + \alpha^2 v c}{c^2 \alpha^2} = \frac{c - v + v}{c} = 1$$

Relativity of simultaneity is thus covariant, at least with respect to different speeds and locations of only two observers.

Lorentz Transformations

Transformation equations transform distance and time coordinates of observers in relative motion to the coordinates of the other observer. Let x, y, z, t be the spacetime coordinates of Observer **A** relatively at rest, and let corresponding coordinates of Observer **B** relatively in motion be x', y', z', t'. Respective origins O and O' of the two coordinate systems overlap at time t = t' = 0, such that time and distance of the events in coordinate system **B**, as perceived by Observer **B**, transform in view of Observer **A**'s perception, and vice versa for perceptions of each other's coordinates to be the same.

First consider a Galilean transformation of coordinates with regard to system **B** moving at velocity v relative to system **A**. Let origins of respective observers **A** and **B** be at the same place during the instant $t_0 = 0$, such that the coordinate distance s system **B** moves at velocity v becomes shorter for the time t by the amount $s^* = vt$. Hence

$$s^* = s - vt$$

The result only assumes the measures of distance and time are relatively the same for all observers.

The event could be in any direction of relative motion. To describe it according to Galilean relativity whereby clocks and distance coordinates are not affected by relative motion, three-dimensional-perpendicular coordinate systems are established. Coordinate system **B** has perpendicular coordinates from origin O at time t are x, y, z, t for both **A** and **B**. For the comparison of relativistic values, coordinates x', y', z', t' are used for **B** and coordinates x, y, z, t for **A**, as designating **B** in relative motion and A relatively at rest.

Lengths y' and z' are perpendicular to the direction of motion and are not contracted by it. Hence, y' = y and z' = z. In the manner of explaining null results of the Michelson-Morley experiment, the distance light actually moves perpendicular to the direction of relative motion is increased by the same mathematical factor the time of a clock in relative motion is increased. The extended time it takes light to propagate farther is thus negated by the slower clock.

In contrast, the x and x' directions of relative motion moving relative to each other in opposite directions are consequential to how distances are determined by each observer. Since Observer **B**'s clock is slow, **B** moves an extended duration and an extended distance for modification of a Galilean transformation by a relativistic factor to be of the form

$$x' = \frac{x - vt}{\sqrt{1 - \beta^2}}$$

As for the transformation of time coordinate, distances coordinates convert into time coordinates and vice versa. With constant light speed as a measure of distance, the respective times t and t' can be replaced with x/c and x'/c. Substituting ct' for x', ct for x, and x/c for t obtains

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \beta^2}}$$

This is the Lorentz transformation that transforms the coordinate time t' of system \mathbf{B} in view of system \mathbf{A} .

The transformations indicate changes in time and distance because of different time and distance for light to move from each position of relative motion. If the object of reference is approaching the observer, distance and time of light speed are subtracted from those of actual time and distance of movement of the object. For objects receding from the observer, time and distance of light speed are added, as v is then of a negative value.

Generally, the transformations apply to different perceptions of events by different observers; not merely the relative motion itself between the two observers. Another observer or object, for instance, can be included in the analysis. The speed of the additional object is determined differently by the two observers, such that transformations apply accordingly. Although they perceive different results, either can still be considered as relatively at rest.

Adding Velocities

Since clocks in relative motion differ, it is not obvious how observers in relative motion determine other velocities, as their calculation requires a theorem for adding velocities. For deriving it, consider inertial systems **A**, **B** and **C**. Consider **A** as relatively at rest, **B** as moving at velocity v_1 relative to **A**, or **C** as moving at velocity $v_2 = x'/t'$ relative to **B**. The sum of velocities v_1 and v_2 is to be determined relative to **A** as velocity $v_{12} = x/t$.

In relating coordinates of **B** to those of **A**, velocity v₂ transforms as

$$v_{2} = \frac{x'}{t'} = \frac{[x - v_{1}t]\sqrt{1 - \frac{v_{1}^{2}}{c^{2}}}}{\left[t - \frac{v_{1}x}{c^{2}}\right]\sqrt{1 - \frac{v_{1}^{2}}{c^{2}}}} = \frac{x - v_{1}t}{t - \frac{v_{1}x}{c^{2}}}$$

Multiplying the first and last sides of the equation by $t - v_1 x/c^2$ obtains

$$v_2\left(t - \frac{v_1 x}{c^2}\right) = x - v_1 t$$
$$v_2 t - \frac{v_1 v_2 x}{c^2} = x - v_1 t$$

Adding v_1v_2x/c^2 and v_1t to both sides of the equation obtains

$$v_1 t + v_2 t = x + \frac{v_1 v_2 x}{c^2}$$
$$t(v_1 + v_2) = x \left(1 + \frac{v_1 v_2}{c^2}\right)$$

Finally, dividing both sides of the equation by t and $1 + v_1v_2/c^2$ obtains

$$\frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{x}{t} = v_{12}$$

This equality represents velocity of C in relation to the coordinate system A.

The formula above is for comparing velocities in the same direction of motion. It is possible to derive a formula for systems moving perpendicular to each other as well, which is actually simpler because of no contraction of length in the perpendicular direction of relative motion. Consider an object rotating perpendicular to the forward direction of relative motion. Because the clock in relative motion is slow, the observer perceives a faster speed of rotation by a relativistic factor. However, the actual speed of rotation is the combined vector product of perpendicular and forward speeds according to the Pythagorean Theorem.

Consider **A** is relatively at rest, **B** as moving along the x-axis at velocity v_1 , and **D** moving along the y'-axis at velocity v_3 . In relating coordinates of **B** to those of **A**, velocity v_3 becomes

$$v_3 = \frac{y'}{t'} = \frac{y\sqrt{1 - \frac{v_1^2}{c^2}}}{t - \frac{v_1x}{c^2}} = \frac{y\sqrt{1 - \frac{v_1^2}{c^2}}}{t\left(1 - \frac{v_1x}{tc^2}\right)}$$

Multiplying by $(1 - v_1x/tc^2)$ and dividing by the square root of $(1 - v_1^2/c^2)$, the left and right sides of the equation become

$$\frac{v_3 \left(1 - \frac{v_1 x}{t c^2}\right)}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{y}{t}$$

Since $x/t = v_1$, y/t becomes

$$\frac{y}{t} = \frac{v_3 \left(1 - \frac{v_1 x}{t c^2}\right)}{\sqrt{1 - \frac{v_1^2}{c^2}}} = \frac{v_3 \left(1 - \frac{v_1^2}{c^2}\right)}{\sqrt{1 - \frac{v_1^2}{c^2}}} = v_3 \sqrt{1 - \frac{v_1^2}{c^2}}$$

This result is only of \mathbf{D} moving along the y-axis relative to \mathbf{A} . The velocity of \mathbf{D} in the actual direction of the xy-plane has not yet been determined. To determine it, the Pythagorean Theorem applies with regard to the speeds in perpendicular directions. The result is

$$v_{13} = \sqrt{\frac{y^2}{t^2} + v_1^2} = \sqrt{v_3^2 \left(1 - \frac{v_1^2}{c^2}\right) + v_1^2} = \sqrt{v_3^2 \alpha_1^2 + v_1^2}$$

The velocity v_{13} is that of system **D** relative to system **A**.

The derivation assumes light speed is the same for all observers, and they are to be consistent in showing light speed added to light speed or any velocity is still light speed c:

$$c_{cc} = \frac{c+c}{1+\frac{cc}{c^2}} = \frac{2c}{1+1} = c$$
$$v_{12} = \frac{v_1+c}{1+\frac{v_1c}{c^2}} = \frac{v_1+c}{1+\frac{v_1}{c}} = \frac{c(v_1+c)}{c+v_1} = c$$
$$v_{12} = \frac{c + v_2}{1 + \frac{cv_2}{c^2}} = \frac{c + v_2}{1 + \frac{v_2}{c}} = \frac{c(c + v_2)}{c + v_2} = c$$
$$v_{13} = \sqrt{v_3^2 \left(1 - \frac{c^2}{c^2}\right) + c^2} = c$$
$$v_{13} = \sqrt{c^2 \left(1 - \frac{v_1^2}{c^2}\right) + v_1^2} = \sqrt{c^2 - v_1^2 + v_1^2} = c$$

Light speed is thus the same in all inertial systems according to the addition of velocities formulas.

The Clock Paradox

Observers **A** and **B** in relative motion time each other's clock as slow, but not if **B** moves away from **A**, reverses direction and then returns to **A**. **B**'s clock is then determined as the slower one by direct comparison. This is the well-known clock paradox: both clocks being perceived slower than the other by condition of covariance, but only one clock is slower by means of direct comparison.

The paradox is explainable because the event is not symmetrical. The traveling observer changes direction; the stay at home observer does not. In order to show the clock paradox does not contradict theory, it is thus only necessary to show how Observer **A**'s clock is slow if Observer **A** accelerates instead of Observer **B**. Instead of Observer **B** changing direction to return to Observer **A**, **A** merely accelerates to catch up with **B**. There is symmetry with regard to either **A** or **B** accelerating: **B** moving away from **A** at velocity v uses force to become relatively at rest with system **A** and uses more force to return to **A** at velocity –v, compares with **A** using the force to become relatively at rest with system **B** and using more force to move at velocity v₁₂ in order to catch up with **B**.

To verify symmetry is conditional, consider **B** moves along the x-axis relative to **A** at velocity v before changing direction along the x-axis at time T to move at velocity v instead of –v. With regard to **A** as relatively at rest, wherefrom **B** moves relative to **A**, the comparison of time recorded by **A**'s and **B**'s clocks during the trip is

$$2T' = \frac{2T}{\alpha}$$

This longer time results from \mathbf{B} moving an extended distance because of a slower clock.

The determination of time for the previous event is simple. However, for the event of **A** being relatively at rest and then moving to catch up with **B**, **A**'s clock keeps two different rates while **B**'s clock remains the same. By the addition of velocities theorem, a change in velocity is not simply from v to 2v; instead, it is from v to v_{12} , which involves a corresponding change in rate of clocks. This condition is thus more complex than the previous one.

The task is nonetheless to determine total time for **A** to catch up with **B** results the same as $2T/\alpha$. In perspective, what is to be determined is

$$T + T'' = \frac{2T}{\alpha^2} = \frac{2T'}{\alpha}$$

Time T is **A**'s time while at absolute rest and T" is **A**'s time while moving at velocity v_{12} . Time T' is **B**'s in relation to T.

B moves relative to **A** in time T the distance X at velocity v_1 . In order to catch up with **B**, **A** accelerates from relatively at rest to velocity v_{12} . The time T" it takes **A** to catch up with **B** is according to the difference in their speed: $v_{12} - v_1$ according to **A**, where v_{12} is $v_2 = v_1$ relative to **B**. Hence

$$v = v_1 = v_2$$

$$v_{12} = \frac{v_1 + v_2}{1 + \beta_1 \beta_2} = \frac{2v}{1 + \beta^2}$$

$$T'' = \frac{X}{v_{12} - v} = \frac{X}{\frac{2v}{1 + \beta^2} - v} = \frac{X(1 + \beta^2)}{2v - v(1 + \beta^2)}$$

$$= \frac{X(1 + \beta^2)}{2v - v - v\beta^2} = \frac{X(1 + \beta^2)}{v - v\beta^2} = \frac{X(1 + \beta^2)}{v(1 - \beta^2)}$$

$$\frac{T(1 + \beta^2)}{1 - \beta^2} = \frac{T(1 + \beta^2)}{\alpha^2}$$

The total time relative to A is

$$T + T'' = T + \frac{T(1+\beta^2)}{\alpha^2} = \frac{T\alpha^2}{\alpha^2} + \frac{T(1+\beta^2)}{\alpha^2}$$
$$= \frac{T(1-\beta^2)}{\alpha^2} + \frac{T(1+\beta^2)}{\alpha^2} = \frac{T-T\beta^2 + T + T\beta^2}{\alpha^2} = \frac{2T}{\alpha^2}$$

However, being T' is slower than T by the factor $1/\alpha$, the total time of the event according to **B** is

$$\frac{2T}{\alpha^2}\alpha = \frac{2T}{\alpha}$$

Determinations of slower times are thus the same for **A** and **B** with regard to symmetry of conditions.

The Doppler Effect

The Doppler Effect, as proposed in 1842 by Christian Doppler (1803-1853), explains why a whistle on a train is of a lower pitch, as when the train recedes from the observer, and of a higher pitch, as when approaching the observer. Explanation is according to the vibration of air molecules forming waves. Lower pitch sound is stretched out weaker waves and higher pitched sound is more compacted stronger waves. Stretching results from recession between the source and observer. Compacting results from an observer and source approaching each other.

These effects are of the general dynamics of ordinary particles as well as for light. Bullets fired from a gun, for instance, are more energetic if the gun firing them is moving towards the target rather than away from it. They also apply to light whether light is either particle or wave in nature. Systems approaching each other naturally receive light signals more rapidly than do systems receding from each other. Such effects result from laws of motion, either Newtonian or of relativity theory.

The task is to illustrate general covariance of observers firing bullets at each other for comparison of results with a specific Doppler effect of light propagation. Accordingly, Observer **A** is relatively at rest at the origin from where Observer **B** moves away at velocity v₁. After time T and the distance X of their separation, as according to **A**, **A** fires a bullet at **B**. After time T' = T/α_1 of their separation, as according to **A**, **B** fires a bullet at **A**. The task is to show the time for receiving a bullet is the same for **A** as it is for **B**.

First, consider a time T_b for **B** to receive a bullet from **A** is time T plus time T_x it takes the bullet moving at velocity v_2 to catch up with **B** moving away from X at velocity v_1 . Hence

$$T_b = T + T_x = \frac{X}{v_1} + \frac{X}{v_2 - v_1} = \frac{x(v_2 - v_1)}{v_1(v_2 - v_1)} + \frac{v_1X}{v_1(v_2 - v_1)}$$
$$= \frac{X(v_2 - v_1) + v_1X}{v_1(v_2 - v_1)} = \frac{v_2X - v_1X + v_1X}{v_1(v_2 - v_1)} = \frac{v_2X}{v_1(v_2 - v_1)}$$

This time is according to the clock of **A**. Because **B**'s clock is slower by the factor $1/\alpha_1$, as to perceive less duration, **B** determines the total time as

$$T_b = \frac{X\alpha_1}{v_1} \left[\frac{v_2}{v_2 - v_1} \right]$$

The task now at hand is to determine the time for **A** to receive a bullet from **B**. Because **B**'s clock is slow by the factor $1/\alpha_1$, the time **B** decides to shoot a bullet at **A**, after **B** passes **A**, is $T' = T/\alpha_1$, at the distance $X' = x/\alpha_1$ between **A** and **B**. The bullet's speed is calculated according to the addition of velocities theorem, whereby v_{12} and v_2 are negatives in value relative to **B** because of **B** receding from **A** towards which the bullet is fired at:

$$-v_{12} = \frac{v_1 + (-v_2)}{1 + \beta_1(-\beta_2)} = \frac{v_1 - v_2}{1 - \beta_1\beta_2}$$

Although **A** receives the bullet as moving in the negative direction, as v_{12} to be a negative velocity, the time X'/ v_{12} for **A** to receive the bullet from **B** is additional to the time X'/ v_1 . It is thus positive. Hence, the time **A** receives the bullet is

$$\begin{split} T_{a} &= T' + T'_{x} = \frac{X'}{v_{1}} + \frac{X'}{v_{12}} = \frac{X}{v_{1}\alpha_{1}} + \frac{X}{v_{12}\alpha_{1}} = \frac{v_{12}X}{v_{1}v_{12}\alpha_{1}} + \frac{v_{1}X}{v_{1}v_{12}\alpha_{1}} \\ &= \frac{v_{12}X + v_{1}X}{v_{1}v_{12}\alpha_{1}} = \frac{X(v_{12} + v_{1})(1 - \beta_{1}\beta_{2})}{v_{1}(v_{2} - v_{1})\alpha_{1}} \\ &= \frac{X}{v_{1}\alpha_{1}} \left[\frac{v_{12}(1 - \beta_{1}\beta_{2}) + v_{1}(1 - \beta_{1}\beta_{2})}{v_{2} - v_{1}} \right] \\ &= \frac{X}{v_{1}\alpha_{1}} \left[\frac{(v_{2} - v_{1}) + v_{1}(1 - \beta_{1}\beta_{2})}{v_{2} - v_{1}} \right] = \frac{X}{v_{1}\alpha_{1}} \left[\frac{v_{2} - v_{1} + v_{1} - v_{1}\beta_{1}\beta_{2}}{v_{2} - v_{1}} \right] \\ &= \frac{X}{v_{1}\alpha_{1}} \left[\frac{v_{2} - v_{1}\beta_{1}\beta_{2}}{v_{2} - v_{1}} \right] = \frac{X}{v_{1}\alpha_{1}} \left[\frac{v_{2} - v_{2}\beta_{1}^{2}}{v_{2} - v_{1}} \right] \\ &= \frac{X\alpha_{1}^{2}}{v_{1}\alpha_{1}} \left[\frac{v_{2}}{v_{2} - v_{1}} \right] = \frac{X\alpha_{1}}{v_{1}\alpha_{1}} \left[\frac{v_{2}}{v_{2} - v_{1}} \right] \end{split}$$

This time being the same as T_b , as perceived by **B**, is thus covariant.

A similar event for light signals is achieved by substituting c for v_2 , and v for v_1 , to obtain

$$\frac{X\alpha}{v} \left[\frac{c}{c-v}\right] = \frac{X\alpha}{v} \left[\frac{1}{1-\beta}\right] = \frac{X}{v} \left[\frac{\alpha}{1-\beta}\right] = \frac{X}{v} \left[\frac{\sqrt{1-\beta^2}}{1-\beta}\right]$$
$$= \frac{X}{v} \left[\frac{\sqrt{1-\beta}\sqrt{1+\beta}}{1-\beta}\right] = \frac{X}{v} \left[\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}\right] = \frac{X}{v} \left[\frac{1+\beta}{1-\beta}\right]^{\frac{1}{2}}$$

This is the time either Observer \mathbf{A} or Observer \mathbf{B} sees the other observer to move the distance X.

The relation holds for any distance X or X', and for any time T_a or T_b . It is thus divisible into any number of distances and times. These times and distances relate to the wave properties of light, wavelength λ and frequency f such that

$$f' = f \left| \frac{1-\beta}{1+\beta} \right|^{\frac{1}{2}} \qquad \lambda' = \lambda \left| \frac{1+\beta}{1-\beta} \right|^{\frac{1}{2}}$$
$$f' = f \left| \frac{1+\beta}{1-\beta} \right|^{\frac{1}{2}} \qquad \lambda' = \lambda \left| \frac{1-\beta}{1+\beta} \right|^{\frac{1}{2}}$$

The equations are relativistic Doppler formulae for comparing light sources and observers as either relatively approaching toward or receding from each other at constant velocity. The results are the same whether light is a wave or a particle.

Constant Speed Change

According to Newtonian Mechanics, acceleration as a change in speed increases without limit. To the contrary, the addition of velocities theorem stipulates light speed is a limit for matter to neither exceed nor even reach. However, applying the theorem to a system that constantly changes speed is complex. A change in speed from two seconds of acceleration, for instance, is simply determined in accordance with the addition of velocities theorem, but because the rate of speed change decreases as speed itself increases, the total time of acceleration entails more entailed calculation.

Speed change itself is defined constant in a manner consistent with the principle of covariance. If system **P** is subsequently coincident with inertial systems **A**, **B**, **C**, etc., as if the change from one inertial system to another is of an equal time interval, and if the speed and direction of the motion of **B** relative to **A** is the same as **C** relative to **B**, and so forth, then the change in speed is constant. If the change in speed from **A** to **B** is from v_1 to v_{12} with regard to spacetime coordinates of **A**, then the speed change from **B** to **C** is also from v_1 to v_{12} with regard to the spacetime coordinates of **A**, the

speed change from **B** to **C** is from v_{12} to v_{13} . There is thus simple progression of **P** changing speed from v_1 to v_{12} to v_{13} etc. However, this progression is not by the same relativistic factor relative to **A**, as due to **A** perceiving a decreased rate of speed change at greater speeds.

Symmetry and asymmetry of relative distance is illustrated according to the following illustration.



Covariance here applies with regard to the symmetry of **A**, **B**, **C**, **P** and **P'** simultaneously meeting wherefrom **B** relatively moves at velocity v away from **A** and at velocity –v away from **C**. Symmetry also occurs with regard to speeds of **P** and **P'** being v relative to **B** at the time they catch up with **B**. However, there is asymmetry between **B** and **A** opposite the asymmetry between **B** and **C**. **B** perceives a different change of speed of either **P** or **P'** as 2v, as from either –v to v or from v to –v, which is a greater change than from 0 to v₁₂. Still, v is the speed of **B** relative to **A** in determining d as the distance separating **B**, **P** and **P'** from **A** at time t. Since the speed of **B** is v relative to either **A** or **C**, and the speed of either **A** or **C** is v relative to **B**, and since the initial and final speeds of **P** and **P'** are also v relative to **B**, **P** and **P'** initially move in opposite direction away from **B** at a speed v relative to **B** to slow to relatively at rest with **B** and then change direction to return to **B** at speed v, as required by covariance.

The asymmetry is only according to the difference of how distance and time of acceleration is determined by **B** instead of either **A** or **C**. Symmetry is also evident of **P** and **P'** meeting with **B** whereby change in speed of both is a difference between zero and v_{12} relative to **A** and **C**. Although it allows a determination of the difference between what **B** determines apart from **A** or **C**'s determination, time rates of **P** and **P'** are still not evident.

A mathematical solution to comparing \mathbf{P} 's clock time to \mathbf{A} 's is not here provided, as it entails a more complicated math than simple algebra. What is explained is the distinction of the calculation of time and distance between relativity theory and Newtonian Mechanics.

The addition of velocities theorem with regard to this relativistic speed change is applicable with regard to a relativistic modification of Newtonian Mechanics in determining the distance that occurs from the constant speed change. A distance d of acceleration relative to \mathbf{A} is $v_a t$, as the time taking \mathbf{P} to catch up with \mathbf{B} moving at average speed v_a away from \mathbf{A} . The distance d in relation to the acceleration of \mathbf{P} derives in the manner

$$d = v_a t = \frac{2v_a t}{1 + \frac{v_a^2}{c^2} + 1 - \frac{v_a^2}{c^2}} = \frac{2v_a t}{1 + \frac{v_a^2}{c^2} + \sqrt{\left[1 - \frac{v_a^2}{c^2}\right]^2}}$$
$$= \frac{2v_a t}{1 + \frac{v_a^2}{c^2} + \sqrt{1 - \frac{2v_a^2}{c^2} + \frac{v_a^4}{c^4}}}$$

$$=\frac{2v_at}{1+\frac{v_a^2}{c^2}+\sqrt{1+\frac{2v_a^2}{c^2}+\frac{v_a^4}{c^4}-\frac{4v_a^2}{c^2}}}=\frac{2v_at}{\left[1+\frac{v_a^2}{c^2}\right]+\sqrt{\left[1+\frac{v_a^2}{c^2}\right]^2-\frac{4v_a^2}{c^2}}}$$

$$=\frac{2v_{a}t}{\left[1+\frac{v_{a}^{2}}{c^{2}}\right]+\left[1+\frac{v_{a}^{2}}{c^{2}}\right]\sqrt{1-\frac{(2v_{a})^{2}}{c^{2}\left[1+\frac{v_{a}^{2}}{c^{2}}\right]^{2}}}}=\frac{v_{12}t}{1+\sqrt{1-\frac{v_{12}^{2}}{c^{2}}}}$$

The speed v_{12} approximates to $2v_a$. However, v_{12} generalizes for any speed v, as the general formula derived verifiably from the Lorentz transformation equations.

Significantly, if v_{12} is extremely small in comparison to light speed c, as for a negligible effect of the relativistic factor such that it approximates as 1, then the above result compares with a Newtonian non-relativistic one as

$$d = v_a t = \frac{1}{2}at^2$$

By Newtonian Mechanics, v_a is the average speed of P from 0 to 2v, but by the addition of velocities theorem the change in speed from v = 0 to v_{12} is slightly less than 2v because of the rate of speed change at relatively higher speeds decreasing. The difference is negligible except for greater changes in speed from 0 to nearly that of light.

Note: The relativistic result was derived from $d = v_a t$, but its result has different interpretation. Although d is the same for either formula, v_{12} does not represent an average velocity. It is comparable to 2v in accordance with the addition of velocities theorem. The distance d thus equates as either the average speed according to Newtonian Mechanics or as the slower 2vt with regard to the addition of velocities theorem, covariance and symmetry. Because v_{12} relative to **A**'s clock rate t is v_1 relative to **B**'s clock rate t', and is $v_0 = 0$ relative to **C**'s clock rate t'', the addition of velocities theorem can also apply in the manner



The progression of v_0 to v_{12} , as for twice v, is thus in ratio to the relativistic factor squared.

MASS-ENERGY DYNAMICS

How masses in relative motion collide, stick together and maintain the same relative mass as if in relative motion is another paradox of relativity theory. The mass in relative motion is relatively greater than if it is relatively at rest, yet the total energy is also conserved by inelastic collision. Because energy is convertible from one form to another, conservation of mass is not required, but it can be conserved with regard to an increase in internal motion of the mass by means of its collision. The internal motion can be transferred from one mass to another as kinetic energy of heat, or it can be emitted radiation. The latter is one example of how mass-energy converts to light-energy.

The condition of a relative increase in mass and no relative increase in mass is consistent in agreeing with observation and the formulation of SRT. An increase in relative mass with an increase in motion is also derivable in a manner consistent with the conservation laws of momentum and mass. The derivation is initially with regard to an inelastic collision between two equal rest masses whereby a resolution of the paradox is explained with regard to how energy of relative motion is maintained either internally as the result of inelastic collision or used to reverse inelastic collision as elastic collision.

As the paradox is with regard to how masses in relative motion collide, stick together and maintain to have the same relative mass while relatively at rest, the resolution of the paradox considers elastic collision as an inelastic one plus its reverse process. Even though the reverse process is opposite to inelastic collision, it differs inasmuch as the exchange of mass occurs from elastic collision. The two conditions are asymmetrical, but the resolution of the clock paradox was also according to the asymmetry of acceleration, and collisions between masses are a means of acceleration as either symmetrical or asymmetrical effects. Asymmetry of elastic and inelastic collision results from the inelastic collision having other options to how a conditional state of equilibrium is restored. It is asymmetrical when kinetic energy of inelastic collision converts to another form instead of the relative motion before the collision by means of elastic collision.

The resolution of the relative mass paradox applies more generally to other phenomena. Relative mass and momentum are conserved with regard to reflection, absorption and emission of light by matter whereby Einstein's mass-energy equation $E = mc^2$ is derived. In addition, a classical concept of kinetic energy is redefined as the difference between rest mass and mass in relative motion. Since kinetic energy reduces by inelastic collision according to Newtonian Mechanics, it converts to another form according to SRT.

Relative Mass

According to special relativity a mass m in relative motion at velocity v in ratio to light speed c is relatively greater than the same mass m_0 relatively at rest:

$$m = \frac{m_0}{\sqrt{1 - \frac{\nu^2}{c^2}}}$$
(1)

Mass-energy according to theory is conserved after collision. If the collision is inelastic, wherefrom relative motion between masses is terminated, then total massenergy remains unchanged if observed from the same reference frame of motion.

Despite this apparent contradiction, an increase in relative mass along with an increase in relative motion is verifiable mathematically according to the conservation principles of mass-energy and momentum. Constant light speed and covariance also apply, as does the addition of velocities theorem.

Relative mass is first distinguished from the rest mass. Let mo be mass relatively at rest with either Observer A or Observer B. Let m be the same quantity mass if moving with **B** at velocity v_1 relative to a positive direction towards **A**. Mass m and mass m_0 become one, say M, relative to A by means of inelastic collision. Conservation of mass along with conservation of total momentum applies according to equations (2) and (3):

$$m + m_o = M \tag{2}$$

$$mv_1 + m_o(0) = mv_1 = Mv_2 \tag{3}$$

Equation (2) illustrates the total mass is the same before and after inelastic collision. Equation (3) is somewhat more complex due to the $m_0(0)$ being at rest with A before collision. Total momentum relative to A before collision is thus only mv₁, but because both masses move at velocity v_2 after inelastic collision, the total momentum relative to \mathbf{A} is also Mv_2 .

Substituting m and m_0 of equation (2) for M in equation (3) allows for a solution of m as

$$mv_1 = (m + m_0)v_2$$
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$$mv_{1} = mv_{2} + m_{0}v_{2}$$

$$mv_{1} - mv_{2} = m_{0}v_{2}$$

$$m(v_{1} - v_{2}) = m_{0}v_{2}$$

$$m = \frac{m_{0}v_{2}}{v_{1} - v_{2}}$$
(4)

The objective is to verify equation (4) equals equation (1):

$$m = \frac{m_0 v_2}{v_1 - v_2} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{5}$$

$$\frac{m}{m_0} = \frac{\nu_2}{\nu_1 - \nu_2} = \frac{1}{\sqrt{1 - \frac{\nu_1^2}{c^2}}} \tag{6}$$

$$\frac{v_2}{v_1 - v_2} = \frac{1}{\sqrt{1 - \frac{v_1^2}{c^2}}} \tag{7}$$

$$\sqrt{1 - \frac{v_1^2}{c^2}} = \frac{v_1 - v_2}{v_2} = \frac{v_1}{v_2} - 1 \tag{8}$$

$$v_2 \sqrt{1 - \frac{v_1^2}{c^2}} = v_1 - v_2 \tag{9}$$

$$v_2^2 \left[1 - \frac{v_1^2}{c^2} \right] = v_1^2 - 2v_1v_2 + v_2^2 \tag{10}$$

$$v_2^2 - \frac{v_1^2 v_2^2}{c^2} = v_1^2 - 2v_1 v_2 + v_2^2 \tag{11}$$

$$-\frac{v_1^2 v_2^2}{c^2} = v_1^2 - 2v_1 v_2 \tag{12}$$

$$2v_1v_2 = v_1^2 + \frac{v_1^2 v_2^2}{c^2} = v_1^2 \left[1 + \frac{v_2^2}{c^2} \right]$$
(13)

$$2v_2 = v_1 \left[1 + \frac{v_2^2}{c^2} \right] \tag{14}$$

$$v_1 = \frac{2v_2}{1 + \frac{v_2^2}{c^2}} \tag{15}$$

Equation (15) is of the form of the addition of velocities theorem.

Equation (15) relates to equation (1) in a manner of the elastic collision being a reversal of inelastic collision. Consider v_1 is the initial velocity of m towards $m_o(0)$; v_2 is the velocity of m_x after inelastic collision. In relation to the addition of velocities theorem, v_1 further pertains to the result of elastic collision as a reversal of inelastic collision from changes in velocities of m_o and m_x . In view of covariance, velocities v_2 of $m_o(0)$ relative to m_x and $-v_2$ of m_x relative to $m_o(0)$ are the same speed. This covariant symmetry further applies to reversals of inelastic collision. The change in speed of $m_o(0)$ from v_2 to 0, with m_x relatively at rest, is from 0 to v_2 , as reversing of inelastic collision. However, with respect to $m_o(0)$ as relatively at rest, it is according to the addition of velocities theorem for twice change of v_2 from 0 and from v_2 .

The conservation of mass in inelastic collision indicates a paradox with regard to the same mass being relatively greater if in relative motion, but the paradox is resolved with regard to conservation of mass only applying as a momentary state of collision. If collisions are inelastic, mass is conserved by internal motion of the system, as heat or some other form of energy. If they are of internal motion, then conservation of mass is maintained, but mass is not conserved as mass if it converts to such electromagnetic energy as light. If the internal energy converts to light energy, then mass is only conserved if some other mass absorbs an equal amount of light energy.

Mass-Energy and Light

Say inelastic collision occurs between two masses for them to become internally the same as they were in relative motion. The law of conservation of energy maintains. For, if collision is inelastic in nature, then motion stops unless it continues as some other form of energy. Internal energy could be stored, for instance, as heat or molecular motion. With elastic collision, the kinetic energy is restored as relative motion of the masses. For a collision to remain inelastic whereby the same mass content of the system remains as it was before collision, the internal energy of motion caused by the collision is spent to maintain in a state of equilibrium with its environment. It could be spent either as the internal motion of molecules of matter or as radiant heat, as a means to maintain thermodynamic equilibrium.

Heat is viewed as a random motion of internal molecules of mass, but conservation of relative mass and momentum applies to molecular motion. The molecules are not able to directly surrender their motion to molecules when they are separated by empty space. However, there is still a possibility to consider of heat being absorbed and emitted as radiant energy, such as is light. Since energy is conserved of masses in collision by them exchanging it, a kinetic energy of relative motion also maintains in some form or another. Einstein agreed. Mass and energy according to SRT are equivalent whereby the mass content of a body is only one of many possible forms of energy as convertible from one form to another.

Consider a form of energy as that of light. Light emitted, absorbed or reflected by matter constitutes pressure, as according to an electromagnetic theory that was derived by Maxwell in the middle of the 19th century, and as was verified by experiment in the year 1890 by Peter Lebedew (1866-1911). Reflection and absorption of light by two lightweight flags, for instance, can cause them to rotate around a tiny pole if they are attached opposite to each other, as by one side of each flag being colored white to reflect light and the other side of each flag being the color black to absorb it. The absorption of light by one flag while the other flag reflects it results in unequal momenta.

Einstein equated the internal energy of mass as a product of mass and light speed squared in the manner

$$E = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}} = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Mass m_0 is the rest mass of m moving at velocity v, and m_0c^2 is the "internal energy constitutive of rest mass". The rest mass in relative motion increased by $m - m_0$ is identified as the kinetic energy potential for relative motion in approximation to kinetic energy of Newtonian Mechanics: $K = (\frac{1}{2})m_0v^2$.

Einstein explained the relation according to a tube with two spheres of mass positioned at each end of the tube. The two spheres are here denoted as A and B. Total mass M is of the tube and spheres considered relatively at rest, except for sphere A having an excess amount of potential energy E_0 . A photon with energy E_0 is emitted from sphere A to sphere B at speed c and momentum E_0/c . The tube of spheres recoils in the opposite direction that speed v and momentum Mv do. Conservation of momentum applies in the manner

$$Mv = \frac{E_0}{c} \tag{1}$$

$$v = \frac{E_0}{Mc} \tag{2}$$

Equation (2) is to be used to substitute values.

Momentum is negated when sphere B absorbs light and passes it onto the tube by means of inelastic collision with momentum being of the same magnitude moving in an opposite direction, but displacement occurs from one sphere to the other during time t of light propagation as

$$x = vt \tag{3}$$

Substituting E_0 /Mc of equation (2) for v of (3) gives

$$x = \frac{E_0 t}{Mc} \tag{4}$$

To restore the tube to its original position, the particle of mass m_0 equal in energy to that surrendered as light by sphere B is transported the distance d back to sphere A. With all other activity balancing out on the transporter's return trip, the transportation causes the tube to move back a distance x to its original position.

Work occurring for the tube to move the distance x equals the product dm_0 in regard to the mass m_0 is a measure of weight. Since the particle and the light are of the same energy, and since the work energy was used by the particle in replacing the light energy back to its original position, which is a process restoring the original position of the tube, the actions equate in the manner

$$xM = dm_0 \tag{5.1}$$

$$x = \frac{dm_0}{M} \tag{5.2}$$

Combining equations (4) and (5.2) gives

$$x = \frac{dm_0}{M} = \frac{E_0 t}{Mc} \tag{6}$$

The last two equalities give

$$\frac{E_0 t}{Mc} = \frac{dm_0}{M}$$

$$E_0 = \frac{Mm_0 cd}{Mt} = \frac{m_0 cd}{t}$$
(7)

However, d is the distance of light propagating from sphere A to sphere B in time t. In other words, d/t is the speed of light. Hence

$$E_0 = m_0 c^2 \tag{8}$$

The last equation is the internal energy constitutive of the rest mass m_0 , not including the energy of relative motion.

Since the mass in relative motion is increased by a factor $(1 - v^2/c^2)^{-1/2}$, the mass-energy equation is

$$E = mc^{2} = \frac{m_{0}c^{2}}{\sqrt{1 - \frac{v^{2}}{c^{2}}}}$$

The energy constitutive of mass is thus internal energy plus external energy of relative motion.

Conserving Light Energy

Since light possesses momentum, its emission and absorption by mass is subject to the conservation laws of momentum and energy. Verification is with regard to mass $m_0 = 1$ with momentum $m_0(0) = 0$ relative to Observer **A** absorbing a photon of light having momentum $m_x c$ such that m_0 and the photon energy move with Observer **B** at velocity .6c relative to Observer **A**. Conservation of momentum with regard to the total mass of both light and matter with respect to **A** is according to the equations

$$m_{x}c + m_{0}(0) = (m_{x} + m_{0}).6c$$

$$m_{x}c = .6m_{x}c + .6m_{0}c$$

$$m_{x} = .6m_{x} + .6m_{0}$$

$$.4m_{x} = .6m_{0}$$

$$m_{x} = \frac{3}{2}m_{0}$$

The total mass with respect to A before and after absorption is thus

$$\frac{3}{2}m_0 + m_0 = \frac{5}{2}m_0$$

Total momentum before collision was (3/2)(1) = 3/2 light speed units. It is also (3/2 + 1)(3/5) = (5/2)(3/5) = 3/2 light speed units after collision. The total momentum is thus conserved with regard to absorption and emission of light by matter.

Note: Since the total mass before and after the collision is simply $m_x + m_0 = 2.5$ units with respect to **A**, it is also conserved, as in accordance with inelastic collision.

For a reverse process of inelastic collision as elastic collision, a photon is emitted from total mass $m_x + m_o$ in the opposite direction at which it was absorbed. Although **B** perceives speed of emission the same as absorption, **A** perceives the emitted photon speed as

$$v_{12} = \frac{2v}{1 + \frac{v^2}{c^2}} = \frac{2(.6c)}{1 + .36} = \frac{15}{17}c$$

The relative mass becomes

$$\frac{m_0}{\sqrt{1 - \left[\frac{15}{17}\right]^2}} = \frac{17}{8}m_0$$

Its momentum becomes

$$\frac{17}{8}m_0 \cdot \frac{15}{17}c = \frac{15}{8}m_0c$$

The photon *mass-inertia* in the opposite direction is

$$m_{xb} = m_x \left| \frac{1 - v_{12}}{1 + v_{12}} \right|^{\frac{1}{2}} = \frac{3}{2} m_0 \left| \frac{c - \frac{15}{17}c}{c + \frac{15}{17}c} \right|^{\frac{1}{2}} = \frac{3}{8} m_0$$

Total mass and total momentum with respect to A are again

$$\frac{17}{8}m_0 + \frac{3}{8}m_0 = \frac{5}{2}m_0$$
$$\frac{17}{8}m_o\frac{15}{17}c - \frac{3}{8}m_0c = \frac{3}{2}m_0c$$

Both mass and momentum of light and matter are thus conserved of elastic collision between light and matter by means of a transfer of mass between light and matter.

Since the total energy of matter relatively at rest equals the product of mass and light speed squared, and since elastic collision between matter and light involves a transfer of mass for its conservation, implications are matter is essentially a collection of light energy in that it converts from a light form to a mass form by means of inertia.

Light Energy and Entropy

Entropy relates as stored energy, such as a potential to enhance energy of light. Different temperatures allow for the release of stored energy. The process is generally complex in nature. Water can boil inside a paper cup at a campfire. The hot flame does not even change the appearance of the cup. However, attempting to put out a hot grease fire on a stove with water will only fuel it. Water thus has capacity for absorbing a tremendous amount of heat as well as for creating it.

Entropy and light energy thus seem to have a common connection for further interrelation. This interrelation is with regard to kinetic mass-energy and the storage potential of light energy. We consider here Boyle's Law and the Ideal Gas Law in contrast to the Stefan-Boltzmann Fourth Power Law. There is further consideration regarding mass-volume ratios determined by Littmann and a possible explanation in accordance with the containment of internal energy by outside force that was proposed by Aspden.

Boyle's Law relates to a constancy of pressure and volume of a gas at a particular temperature. According to an Ideal Gas Law, different values for different temperature are proportional to temperature, if not for such other factors as electromagnetic light energy and entropy. As with light intensity, a Fourth Power Law was derived by Boltzmann whereby intensity change is proportional to the fourth power change in temperature. Similarly, a nuclear density of the hydrogen atom, of one proton, to its outer electron-density is to the fourth power. However, Littmann determined mass-volume ratios of the proton and electron, whereby volumes are only to the third power. The fourth power condition could be the force of containment that was part of Aspden's in deriving the ratio of proton mass to electron mass, as it is part of the derivation of the Stefan-Boltzmann fourth power law as well.

In the Boltzmann derivation, light intensity is related to energy density, which further relates to the force containing it, which is similar to how the proton-electron ratio was derived by Aspden. As to how the fourth power condition manifests, consider light to be wave energy in reverse proportion to wavelength and in proportion to frequency. Compare a volume of space containing twice longer waves of one-half frequency whereby wave-energy is one-eighth the density. The ratio of temperatures could be to the fourth power of light intensity in that sixteen times more light energy is needed in order to double its temperature energy.

Explaining Gravity

By theory, any particle moving at light speed is massless. A conversion of light energy to mass energy is by light absorbed by mass. If the process is of inelastic collision, then total mass in increased unless there is recycling of the light by the total process. The process could somehow include emission of radiant energy as the release of work energy resulting indirectly from the increase in molecular motion caused by mass absorbing light. However, an elastic collision between light and mass also increases molecular motion for more release of work energy. The recycling process need work both ways. A release of additional radiation, for instance, could result in more capacity of mass to absorb other light energy.

Other means of converting light energy to mass energy are by gravity and the Higgs Mechanism. According to Quantum Theory, there are virtual particles such as massless Higgs particles moving at light speed that acquire mass in slowing through a Higgs field, as a virtual field of potential energy. Such a mechanism complies with conservation of energy inasmuch as it is neither created nor destroyed but only converted into another form.

The Higgs Mechanism succeeded as part of the unification of three of four fundamental forces of nature: the strong nuclear force, the weak force of subatomic particle decay and electromagnetism. However, although the Higgs mechanism explains creations of mass on the atomic scale, it does not explain the general creation of mass, and the gravitational force is still not a part of the unification.

According to the principle of equivalence the forces of relative motion apply to gravity as well. To explain gravity as such, consider it as a particular field created by the presence of mass. The Higgs particle is very massive but extremely short lived in creating a vacuum effect for a boundary condition. Consider a more general vacuum effect for matter as its gravitational field. Matter converts to gravitational radiation in creating a vacuum effect in the wake of the emitted radiation. If the amount of gravitational radiation is in direct proportion to the amount of mass, and if two emissions occur from the same place, and they emit in exact opposite directions according to the Doppler principle, then the dynamics of the field maintain consistent with the dynamics of relative motion according to an equivalence of inertial and gravitational mass.

What is still needed of this explanation of gravity is to explain further how it complies with conservation of energy. How, for instance, does mass continue to create gravitational radiation without using up the energy of the field? The action could be similar to an inelastic collision whereby energy is internally converted into another form. Virtual vacuum energy converts into mass as mass converts into gravitational radiation, which need also convert into another form for a recycling process. It thus has a finite range for it to convert from massless particles to virtual particles to maintain the recycling process.

The recycling process along with the virtual field of energy could also be relative in their means of supporting conservation of energy, as there is also another condition of relativity to consider about the change in total mass of the universe at large in the view of an observer's changed state resulting say from collision with other mass. Since measure of distance and time changes with a change in relative motion, another mass apart from the collision relative to the new state of motion could appear in violation of the conservation of energy law.

As to why mass-energy conservation does not necessarily result from a change in speed of the observer, it is a more complex issue that includes the effects of gravity and the probability condition of quantum physics in relating to the nature of the universe at large. A change within an isolated system of an observer cannot maintain energy conservation unless relativistic decreases in speed balance with relativistic increases in speed. This balance is inconceivable if the initial-average-speed is greater than the final-average-speed. Observation of the same speed maintains only if systems are not truly isolated perceptual wise. For instance, the universe as finite might only be perceived as such. If local states of observers change, a change in the universe could change as well, as according to complete mass-energy conservation of a virtual energy field of quantum physics and its probability condition of observing indirect effects of virtual particles.

Such a condition of energy conservation is not required since mass that does not accelerate is not a symmetrical condition of accelerating mass, but the gravitational state of the universe at large could still be consequential. A changed state of the observer thus corresponds to a perceptual state change in the universe at large.

THE RELATIVITY OF GRAVITY

After modifying Galilean relativity for it to comply with relative spacetime instead of absolute space and absolute time, Einstein focused on Newton's theory of gravity for its compliance with relative spacetime as well. He considered gravity as analogous to relative motion in compliance with an equivalence principle fundamental to both Newtonian Mechanics and relativity theory, whereby gravitational mass and inertial mass are the same thing. A change in motion of mass thus occurs either from a collision with another mass or by the gravity of the other mass. A change in velocity is double that of another by collision with or of gravity of the other mass if the other mass is twice as much.

Another principle Einstein used in relating gravity to relative motion is similar to one Copernicus previously invoked to explain our unawareness of Earth's orbital motion. Copernicus realized that we are not internally aware of Earth's motion around the sun because of us being uniform with Earth's motion around the sun. Einstein also reasoned we are internally unaware of falling freely by gravity because, as had been determined by Galileo, all mass falls at the same rate through a vacuum towards Earth's center. He also assumed light is gravitated along with mass for no internal awareness of change, which is consistent with describing spacetime according to constant light speed. However, gravity is inhomogeneous by nature, as to complicate the equivalence principle in that parts of a system gravitate towards a center of mass according to various distances and different directions of free fall.

Einstein equated free fall with inertial motion inasmuch as there is no internal awareness of either one, but free fall of Earth with its moon can be felt by way of ocean tides, as parts of Earth closer to the moon gravitating more towards it. There is also a tendency of mass to gravitate towards a common center instead of in parallel direction. Objects inside a container in free fall thus tend to converge towards a mass center. Furthermore, what is opposite to falling inward is a curvature of orbital speed. An increase in orbital speed at the same radial distance results in a straighter orbital path, as for moving farther contrary to falling at the same rate. A ball moving faster also moves potentially farther before lowering to the ground. The greater speed of light perpendicular to the radial distance similarly moves straighter distances than does slower mass falling at the same rate towards the center of mass. There is thus a greater tendency of orbital escape by the speedier light.

Since constant light speed is the founding principle of SRT, and since light is assumed to gravitate toward matter, Einstein relegated the validity of SRT as a special case, applicable as far as tidal effects are negligible. They are negligible, for instance, inasmuch as particular parts of the gravitational field are perceivable as homogeneous whereby different distances are too short to determine their differences of gravitational effect. Einstein then opted for a geometrical description of gravity in accordance with spacetime curvature due to the presence of mass.

Even though Einstein opted to stipulate SRT is valid only as a special case, it can still be considered as an integral part of GRT inasmuch as the latter is a modification of the former whereby analogies of such relativistic effects as retardation of clocks, contraction of length, occur according to both SRT and GRT. Moreover, even though GRT describes spacetime as curved, whereby light speed varies, SRT and GRT still interrelate by geometrical invariance, particularly in view of a Schwarzschild Metric derived by Karl Schwarzschild (1873-1916) and published in 1916.

Invariance

The Pythagorean Theorem is useful in surveying when the line of sight for a direct measure of distance is blocked by an obstacle. A right triangle hypotenuse provides an invariant for mapping an alternative direction as a detour, with the hypotenuse as determinable for distance. Because the two other legs can extend perpendicular to each other in any directions from opposite ends of the hypotenuse, their relation to the hypotenuse determines an invariant for all right triangles formed from it. As illustrated below, the invariant is of the form $C^2 = A^2 + B^2 = A'^2 + B'^2$. A² differs from A'² and B² differs from B'², but the total area is the same, as C², for both right triangles.



This invariance also applies to higher dimensions, as according to the rectangular box below. Length OP is the hypotenuse of a right triangle with its other sides as x and y. Length OQ is also a hypotenuse of a right triangle with its other sides as OP and z. Length OQ is thus according to the legs of two right triangles of sides x, y and z.



Physics includes, among other things, time and motion, and with SRT as first formulated by Einstein and then given geometrical interpretation by Hermann Minkowski (1864-1909), time becomes a fourth-dimension of the spacetime interval s in the manner

$$s^{2} = x^{2} + y^{2} + z^{2} - c^{2}t^{2} = x'^{2} + y'^{2} + z'^{2} - c^{2}t'^{2}$$

The letter s denotes the interval, as to determine a value of a variable of one coordinate system if other values of two coordinate systems are known.

Invariance of the interval derives from the Lorentz transformations:

$$x' = \frac{x - vt}{\sqrt{1 - \beta^2}}$$
, $y' = y$, $z' = z$, $t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \beta^2}}$

For the derivation, time coordinates convert into distance coordinates in the manner

$$ct' = \frac{ct - \beta x}{\sqrt{1 - \beta^2}}$$

The coordinates are then added and subtracted in the manner

$$ct' + x' = \frac{ct - \beta x}{\sqrt{1 - \beta^2}} + \frac{x - vt}{\sqrt{1 - \beta^2}} = \frac{ct - vt + x - \beta x}{\sqrt{1 - \beta^2}} = \frac{ct(1 - \beta) + x(1 - \beta)}{\sqrt{1 - \beta^2}}$$

$$ct' - x' = \frac{ct - \beta x}{\sqrt{1 - \beta^2}} - \frac{x - vt}{\sqrt{1 - \beta^2}} = \frac{ct + vt - x - \beta x}{\sqrt{1 - \beta^2}} = \frac{ct(1 + \beta) - x(1 + \beta)}{\sqrt{1 - \beta^2}}$$

The product (t' + x')(t' - x') gives

$$t^{2} - x^{2} = \left[\frac{ct(1-\beta) + x(1-\beta)}{\sqrt{1-\beta^{2}}}\right] \cdot \left[\frac{ct(1+\beta) - x(1+\beta)}{\sqrt{1-\beta^{2}}}\right]$$
$$= \frac{c^{2}t^{2}(1-\beta^{2}) - ctx(1-\beta^{2}) + cxt(1-\beta^{2}) - x^{2}(1-\beta^{2})}{1-\beta^{2}} = c^{2}t^{2} - x^{2}$$

Hence, invariance of the interval is of the form

$$s^{2} = c^{2}t'^{2} - x'^{2} = c^{2}t^{2} - x^{2}$$
$$\frac{s^{2}}{c^{2}} = t'^{2} - \frac{x'^{2}}{c^{2}} = t^{2} - \frac{x^{2}}{c^{2}}$$
$$\frac{s^{2}}{t^{2}} = c^{2} - \frac{x^{2}}{t^{2}} = c^{2} - v^{2}$$
$$\frac{s^{2}}{t'^{2}} = c^{2} - \frac{x'^{2}}{t'^{2}} = c^{2} - v^{2}$$

Significantly, invariance of the interval is simpler and more convenient for relating different phenomena. In the same manner of relating spacetime coordinates, for instance, momentum and energy are invariant according to the equations

$$\frac{s^2}{c^2} = P_x^2 + P_y^2 + P_z^2 - \frac{E^2}{c^2} = P_{x'}^2 + P_{y'}^2 + P_{z'}^2 - \frac{E'^2}{c^2}$$
$$s^2 = P_x^2 c^2 + P_y^2 c^2 + P_z^2 c^2 - E^2 = P_{x'}^2 c^2 + P_{y'}^2 c^2 + P_{z'}^2 c^2 - E'^2$$

Total energy or momentum of systems thus calculates by knowing variables of energies or momenta of one system in comparison to another.

Invariance of Spacetime Curvature

The conception of Newton's inverse-square-law for gravity as acting at a distance is contradistinctive with general relativity's explanation of gravity according to spacetime curvature. A change in distance, according to action at a distance, causes instantaneous change in gravitational effect. However, transport of information faster than light speed is contrary to the condition of special relativity. Einstein thus generalized covariance of spacetime for it to comply with gravitational effect, as general covariance, which maintains invariance of the laws of physic in being the same for all reference frames. A field theory refers to magnitudes in space and time differing according to temperature, gravitational effect and so forth. The convenient way of relating them is by a mathematical matrix, and the simplest form of Einstein's field equation is a tensor matrix for spacetime curvature, as

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The left side of the equation is Einstein's tensor matrix and the right side is a stressenergy tensor matrix in compliance with the Minkowski spacetime invariance for momentum and energy. The $8\pi G/c^4$ is an Einstein constant including Newton's constant G and light speed c. The $4\pi G$ is in relation to surface area of a sphere. The dimensionality (cubic centimeters per second squared and per gram) of 2G relates as a distance r multiplied by velocity v squared per mass m: v²r/m. Divided by c⁴ it becomes dimensionally as per centripetal force: r/mv². Multiplied by an energy potential mv² per volume, as proportional to r³, its dimensionality is per area, r⁻². If r⁻² is divided by c² to become t⁻², it then relates as energy instead of as momentum. Moreover, optional is an inverse of centripetal force coupled with the stress momentum-energy tensor $T_{\mu\nu}$ for determining the spacetime curvature of a gravitational field according to surface area of a volume space per pressure, as pounds per surface area and as the weight of gravitational force.

The subscripts $\mu\nu$ of the matrixes are similar to spacetime coordinates x, y, z, t of relative motion, but they apply more specifically to Riemannian curved space apart from Euclidian spacetime coordinates, and they are also more complex in describing positions in four dimensional spacetime instead of only coordinate directions for relative motion. The matrix thus allows for a more descriptive geometry according to the distribution of mass-energy.

Einstein used Riemann curvature tensor calculus to describe spacetime curvature of general relativity. A tensor of the first rank is a scalar, which is a magnitude of something such as temperature or mass quantity that can be described by a single number, as a magnitude. A tensor of the second rank is a vector, as for including a direction along with magnitude. Higher ranked tensors are more complex. The Riemann tensor, for instance, describes the shortest path along a curved surface according to a geodesic matrix $g_{\mu\nu}$.

The spacetime curvature in general relativity includes the $g_{\mu\nu}$, as to be determined by the stress energy tensor. By substitution, the Einstein tensor on the left side of the equations becomes

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

The Riemann Curvature Tensor matrix $R_{\mu\nu}$ along with the scalar magnitude R and geodesic curvature tensor matrix $g_{\mu\nu}$ are thus determined according to the value of the momentum-energy tensor matrix $T_{\mu\nu}$.

The Schwartzschild Metric

 $T_{\mu\nu}$, as a stress momentum-energy tensor, is an invariant for describing the effects of all forms of energy. Its applications are complex and there are various solutions as to whether the field rotates and so on. The tensor itself is according to conservation of energy in being consistent with the Lorentz invariance previously derived by Minkowski spacetime in relating invariance of internal mass-energy and momentum.

The Schwartzschild Metric with a non-rotational condition in relation to special relativity and a homogeneous perception of the gravitational field is the simplest solution, which includes the Newtonian form of gravitational potential for modification:

$$ds^{2} = \left[1 - \frac{2GM}{rc^{2}}\right]c^{2}dt^{2} - \left[1 - \frac{2GM}{rc^{2}}\right]^{-1}dr^{2} - r^{2}d\theta^{2} - (sin^{2}\theta)d\varphi^{2}$$

The last two terms of the metric with trigonometric quantities θ and φ refer to polar coordinates in place of the perpendicular coordinates y and z of flat spacetime. Flat spacetime refers to gravitationally free space wherefrom the conditions of SRT apply. They apply to the coordinate lengths of time dt and distance dr as infinitesimal increments. Increments dt and dr interrelate by the metric to equate in terms of an invariant as the interval ds.

The increments dt and dr relate to gravitational homogeneity. Gravity is more the same in effect at less difference in radial distance from a mass surface. Gravity thus converges to a state of homogeneity of infinitesimal differences between radial distance and time.

Note: dt and dr, as increments of relatively homogeneous gravitational spacetime in analogy to relative motion, are conditional to both acceleration and non-acceleration. For instance, in free fall there is no internal awareness of acceleration in the sense the outside view of the world is hidden, but an observer in free fall seeing the outside world, as it is, does perceive a change in velocity in further view of the rest of the world. This external view is also analyzable as is the internal view. Similarly, even though acceleration, as explained according to the clock paradox, is an asymmetrical condition of the slower clock being the one that changes direction, covariance still complies in the sense the laws of physics are the same for all observers. The principle is maintained in general relativity as general covariance.

Incremental differences allow for Newtonian Mechanics to apply as an initial state that is modified according to the relativity of spacetime. The modification is according to equivalence of inertial and gravitational mass whereby either $(1 - v^2/c^2)$

or $(1 - 2GM/rc^2)$ is referred to as a relativistic factor. A meaning of relative is 'comparable to' and the relativistic factors thus refer to effects as comparable to relatively at rest in gravitational free space.

Variable Light Speed

A distinction of the form of the Schwartzschild Metric from that of Lorentz Invariance is with regard to acceleration whereby the speed of light is derived as slower within the gravitational field. However, it is still possible to explain spacetime events according to either constant or variable light speed. The choice is arbitrary, but maintaining constancy of light speed is more consistent with the spacetime of special relativity.

According to a principle of simultaneity of special relativity, spacetime events are seen according to constant light speed. If the event is a particle of light emitted and absorbed, then its own difference is zero, such that the Schwartzschild Metric in the absence of polar coordinates becomes

$$0 = c^{2}dt^{2} \left[1 - \frac{2GM}{rc^{2}} \right] - dr^{2} \left[1 - \frac{2GM}{rc^{2}} \right]^{-1}$$
$$dr^{2} \left[1 - \frac{2GM}{rc^{2}} \right]^{-1} = c^{2}dt^{2} \left[1 - \frac{2GM}{rc^{2}} \right]$$
$$\frac{dr^{2}}{dt^{2}} = c^{2} \left[1 - \frac{2GM}{rc^{2}} \right]^{2}$$
$$\frac{dr}{dt} = c \left[1 - \frac{2GM}{rc^{2}} \right] = c'$$

The relative speed of light in a gravitational field is thus less than unity.

Although light speed is interpreted as variable in accordance with the Schwartzschild Metric, it is still arbitrary in that the metric also equates with Lorentz invariance. The similarity of Schwartzschild and Lorentz invariance is shown by substituting c' for c in the manner

$$ds^{2} = c^{2}dt^{2} \left[1 - \frac{2GM}{rc^{2}} \right] - dr^{2} \left[1 - \frac{2GM}{rc^{2}} \right]^{-1}$$
$$= c'^{2}dt^{2} \left[1 - \frac{2GM}{rc^{2}} \right] \left[1 - \frac{2GM}{rc^{2}} \right]^{-2} - dr^{2} \left[1 - \frac{2GM}{rc^{2}} \right]^{-1}$$

$$= c'^{2} dt^{2} \left[1 - \frac{2GM}{rc^{2}} \right]^{-1} - dr^{2} \left[1 - \frac{2GM}{rc^{2}} \right]^{-1}$$

Note: The value of c within the relativistic factor need not change because the changes in speed of the escape velocity of the field and light are both of the same ratio as determined by the same relativistic factor squared: $v'^2/c'^2 = v^2/c^2$.

Light speed within a gravitational field is actually perceived as constant by local observes in the field. For understanding how this perception occurs consider the moon's orbit about Earth is in effect a natural clock. The clock is relatively slower within the sun's gravitational field because of the moon's orbit around Earth is slower, as slower light and matter are both according to the relativistic factor squared according to the Schwartzschild Metric. For earthlings, however, the slowing of a clock by the sun's gravitational field is nullified by a slowing of matter and light according to the relativistic factor squared. However, an analogy more consistent with how relativistic effects of relative motion are explained according to special relativity would include a contraction of length in the direction of relative motion.

The more complete explanation of why an observer in relative motion determines the same constant light speed includes contraction of length by the relativistic factor. Longer distances light and objects move according to the system in relative motion are nullified by a contraction of length in the direction of relative motion as well as its slower clocks. In being consistent, there could also be a radial contraction of length in a gravitational field. The natural clock of Earth's moon and other local mechanical clocks within the sun's field thus have relatively shorter orbital distances for them to be only slower by the relativistic factor instead of the relativistic factor squared.

All in all, the slowness of the moon as a natural clock is nullified by the slowness of an observer's mechanical clock. A shorter radial distance of the moon from Earth is likewise nullified because of a shorter measuring by the Earth observer. As with regard to observers relatively at rest in gravitational free space, relativistic contraction of radial distance nullifies the moon from having a slower orbital period by the relativistic factor squared; it is slower only by a relativistic factor instead. The nullification is thus more complex, but it is still analogically consistent with special relativity.

Special and General Relativity Analogies

Spacetime described according to variable light speed is more complex than according to constant light speed. Gravitational analogies of relativistic effects are thus to be explained in the simpler manner according to constant light speed. Instead of variable light speed in the gravitational field, consider spacetime as relatively expanded in analogy to observers in relative motion determining relative distance with their slower clocks. Observers in relative motion determine each other's clock as slow. The coordinate distance each observer moves relative to the other is thus longer than the other because of the extended time of each slower clock perceived by the other observer presumed to be relatively at rest. An observer at rest in gravitational free space likewise determines the gravitational spacetime as relatively expanded according constant light speed and clocks interpreted as slower in the field. However, a local observer in a field of gravity perceives the exact opposite effect of spacetime outside the field than what observers outside the field perceive of the field, as contrary to the observers in relative motion both perceiving the other observer's clock as slow.

The opposite effect condition is analogous to how the clock paradox is explained according to asymmetry. The clock changing direction is the one determined as slow. Similarly, clocks within gravitational fields are the ones determined as slow in comparison to clocks relatively at rest in gravitational free space due to asymmetry of gravitational acceleration. Thus, contrary to internal effects of the free fall of orbiting the sun, observers on the Earth's surface resisting free fall towards its center determine the same light speed as if in gravitational free space, but light from non-local sources outside the field of gravity is perceived differently according to different radial distance of the light source from both the observer and the Earth's center of gravity. Earth observers with slower clocks perceive the events in gravitational free space as occurring in less time, as by either shorter orbital distance or faster orbital speed.

As early as 1906, Einstein had similarly considered effects of gravity as analogical to those of relative motion regarding equivalence of inertial and gravitational mass. By 1911, he concluded the light spectrum blueshifts when entering the gravitational field and redshifts when leaving it. The shifts increase with time and distance in analogy to either increasing redshift or blueshift seen by an observer who increases speed while either receding from or approaching a light source. The Doppler Effect thus increases with time and distance of acceleration by either gravity or another kind of force.

Such red and blue shifts in light spectrum are explainable in a manner consistent with conservation of energy whereby light waves contract by the relativistic factor squared, whereas radial contraction of matter occurs only by the relativistic factor. Light speed equates as the product of wavelength and frequency. Shorter light waves thus have greater frequency, which also equates as greater energy. However, light also has kinetic energy of relative motion. It being slower in a gravitational field by a relativistic factor squared nullifies the greater frequency of light by a relativistic factor squared. There is thus a natural mechanism for conservation of energy. However, Doppler effects need also be explained in accordance with this nullification of effect.

A relativistic contraction of light that slows by a same relativistic factor squared nullifies a change in frequency. Likewise, a slower clock and shorter radial distance by the same relativistic factor nullifies a change in frequency. However, relative motion as orbital motion is also slowed by the relativistic factor squared, but orbital distance is only shorter by the relativistic factor, not squared. If light were only contracted by a relativistic factor, instead of a factor squared, then a complete nullification would occur with regard to the increase in frequency of light, but it would also nullify the explanation of both conservation of energy and observation according to Doppler effect. Contraction by the relativistic factor squared of light thus appears required.

Why, then, is light contracted by the relativistic factor squared whereas matter is only contracted by the relativistic factor? It is because the relative motion itself contributes to the effect.

Actually, the nullifications of effects apply to observers in free fall who are contracted by both the gravitational field and their relative motion, as is light. The observers on the surface of a large mass resisting free fall are only contracted by a relativistic factor. Conservation of energy is thus maintained as observed.

The contracted wavelength is according to a relativistic factor squared, being due to both a relativistic inertia of spacetime in the gravitational field and acceleration of relative motion by gravity. The spacetime inertia is also what slows light and relative motion in the field. It and an increase in speed towards a center of mass are both according to the same relativistic factor, as to comply with the momentumenergy stress conditional to gravitational spacetime consistent with conservation of energy.

Conservation of energy is also illustrated in considering a gravitational field as relatively expanded instead of light speed being slower in it. There is analogy of relativistic increase in mass and energy. Mass in relative motion is perceived as relatively greater than if relatively at rest, and spacetime of a gravitational field is relatively increased in place of decreased light speed and slower motion in the field. In view of Newtonian Mechanics, orbital speed is in ratio to mass and orbital distance. Twice the mass and twice the orbital distance render the same orbital speed. In effect, the orbital speed of the moon around Earth is seen by an observer relatively at rest in gravitational free space as either slower or of the same speed. The slower orbital speed by a relativistic factor is according to the combination of slower light speed by a relativistic factor squared and the contraction of orbital distance by a relativistic factor. However, for constant light speed of larger spacetime, the orbital speed is then in accordance with an increase in orbital distance along with an increase in relative mass, as both according to the relativistic factor. The increase in relative mass entering the gravitational field is consistent with a greater relative mass in relative motion.

Relativistic analogies of general and special relativity thus comply with conservation of energy in that energy increases and/or decreases along with relative motion. If the relative motion changes, then a change in relativistic effect occurs as well. As with the rate of acceleration, opposite effects occur to an observer changing relative to the other.

The Upper Limit

Constant light speed according to special relativity is an upper limit for matter to neither equal nor exceed. A gravitational analogy is with regard to the escape velocity of the gravitational field. In agreement, Einstein argued against the existence of black holes and singularities, but they have become accepted among established physics. The black hole has been modified for it to emit Hawking radiation, but the singularity aspect of it remains part of accepted theory whereby interpretation of the limiting aspect of light speed with regard to gravity allows for conditions of infinite mass-energy density within an infinitesimal space of no definable volume.

Infinity is generally a mathematical problem in relating laws of physics to the observable world. It was a problem relating the thermal light energy (ultra violet catastrophe) until the establishment of the quantum. It continued to be a problem with quantum physics until the principle of renormalization was applied. In relation to the quantum, there is a problem with special relativity stipulating no information of events transmit faster than light speed. Consider the Newtonian definition of acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

The acceleration is a change in velocity v per change in time t, or a circular speed v of radius r. The acceleration can approach infinity if r can approach zero. Moreover, if the change in v occurs simultaneously to a given quantity of mass, then change per distance occurs instantaneously contrary to special relativity stipulating no information can be transmitted faster than light. The discreet unit as the quantum thus appears conditional to relative motion of Newtonian Mechanics in its modified relativistic form.

Is renormalization conditional to relativity theory as well? It could be if $r/t \le c$ is truly stipulating. An infinite speed of c is otherwise allowed for a zero value of t. However, with stipulation that a collision of masses needs to be such that there is no simultaneous action beyond immediate contact there is a quantum condition of probability allowing for virtual effect. Immediate contact is thus of zero extended distance. More reasonable is minimum amount of distance contacted as an unobservable measure that is similar to a virtual condition of quantum physics. The renormalization of infinities of quantum physics is thus a distinction whereby a virtual particle is a mathematical result of otherwise predictive failures of mathematical theory. However, renormalization of relativity theory has not been accepted because of its infinity implications interpreted as a singularity instead.

A mathematical singularity is defined as two terms of an equation, one indicating zero magnitude and the other infinity. The singularity is indicated with the Schwartzschild Metric with the possibility of a gravitational escape speed as that of light. If the escape speed is c, then the ratio of escape speed to light speed is one. Since the relativistic factor is the number one divided by the ratio of an escape speed squared to light speed squared, whereby one minus one is zero, increment dr is divided by zero to indicate mathematical infinity, which is undefined in terms of the observable world, and increment c(dt) multiplied by zero is zero.

An interpretation of the singularity according to the theory that the universe is finite and expanding is that it indicates infinite mass-energy density instead of infinite mass-energy itself. If the origin of the mass-energy of our universe is finite, then there is no violation of energy conservation in that it was finite in the beginning as it is finite now. However, a special relativity analogy indicates a misinterpretation of the singularity.

In accordance with the addition of velocities theorem of special relativity, the speed of light is unattainable of matter by means of collision with light or other matter. Accordingly, there is a decrease in length and increase in mass-energy with an increase in speed. At light speed, mass would thus become infinite mass-energy of an infinitesimal size. In so doing, the mass absorbs mass-energy from whatever accelerates the mass. It thus takes an infinite amount of mass-energy to accelerate mass to light speed.

The Black Hole is similarly controversial. At a particular radial distance is the event horizon whereby the escape speed being that of light is unobtainable because of light speed then becoming zero.

A Schwarzschild Metric interpretation of its singularity is that of the possible existence of unlimited mass-energy density contained within a non-volume space. However, Stephan Hawking challenged this assumption as inconsistent with the laws of thermodynamics, and he redefined the Black Hole for it to emit Hawking radiation.

There is according to special relativity an increase in relative mass with an increase in relative motion. The increase in mass-energy is further shown to be conserved by an exchange of energy between systems. For an analogy, a relative increase in relative mass of a gravitational field, in addition to mass entering into the field, should constitute work energy spent regarding an increase in gravitational force from an increase in relative mass density. As the field increases in mass, it should radiate energy. Moreover, the ratio of radiation to mass should increase for greater escape velocities such that the radiation escaping for an escape velocity approaching light speed should equal that of the additional mass. An addition to the gravitational potentials theorem is thus indicated as analogous to the addition of velocities theorem.

Einstein himself originally did not believe the singularity was real, and he argued against the existence of black holes. Later, in the early 1970s, the condition of the black hole was modified by Stephen Hawking for it to emit Hawking radiation. In 1972, Jacob Bekenstein (1947-2015) generalized the second law of thermodynamics in proposing that black holes should radiate energy in order for them to increase their entropy. Hawking also proposed, in 1970, the event horizons should increase in radial distance, which needs to occur for an increase in mass, but he first rejected Bekenstein's claim of entropy, assuming it need not apply to the entire universe as an isolated system not causing a change to another system, but, in 1974, he considered black holes of higher temperature than the radiation they absorb do violate the second law of thermodynamics. Hawking, in explaining how black holes radiate, applied a probability condition of quantum mechanics to allow the probability of a light particle inside a black hole to exist outside of it as well.

The Cosmological Constant

Einstein who opposed interpretation of the Schwartzschild solution as indicative of a singularity further believed our universe is finite and static. A condition for it to be as such is that it needs a sufficient amount of mass to contain itself. Too much mass would cause it to contract and too little mass would allow it to fly apart. Instead of the Cosmological Principle, the means to counter these two possibilities, he believed, was a repulsive force counter to gravity, which could be represented as a Cosmological Constant that can be inserted into his field equations for a modified form in the manner

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

However, the Cosmological Principle was accepted instead.

The addition of constant Λ is a theoretical possibility inasmuch as the mathematical process of integration allows it. The integration is the reverse process of differentiation whereby exponent powers become reduced in the manner x³ becomes 2x², x² becomes 1x¹, and x becomes 0x⁰ = 0. Because a differentiation eliminates the constants, they become unknowns with regard to the reverse process of integration.

Alexander Friedmann (1888-1925) examined Einstein's field equations and asserted they indicate the finite-static-universe is unstable even with the cosmological constant. The discovery of a redshift in more distant starlight further indicated the universe is expanding. Einstein concurred, referring to his insertion of the cosmological constant as his biggest blunder, but it has of late been reconsidered.

The cosmological constant of general relativity was generally dismissed by physicists, as equaling zero, until 1995 when astronomical observation of redshift data indicated the expansion of the universe has not been constant, as to have increased its rate of expansion. For a possible explanation of this increased expansion rate, the cosmological constant has been reconsidered along with assuming the existence of dark energy as a repulsive force, but a repulsive force of general relativity has not been determined in accordance with empirical data and theory to be consistent with the needed energy for increasing the expansion rate. So far, the required quantum vacuum energy is about 10¹²² times greater than what the cosmological constant provides.

The Cosmological Principle and the Cosmological Constant are both a complication to be resolved for how an expanding universe is explained in a consistent manner.

A Large-Scale Universe of Homogeneity

According to special relativity, mass cannot obtain light speed. If it did, then it would increase to infinite mass. Contrary to this condition is one of general relativity whereby light speed squared is obtainable as a gravitational potential. No addition of gravitational potentials theorem in analogy to the addition of velocities theorem is thereby applicable unless the condition of homogeneity somehow allows it.

Homogeneity not only occurs on an infinitesimal scale; it occurs on a large cosmic scale as well. The reason it occurs is because of nullification of spacetime curvature, or gravitational force, between mass. The gravitational force at the center of Earth, for instance, is zero. If all mass in the universe were to be distributed evenly throughout it, then neither gravitational force nor mass would exist, as in compliance with the principle of equivalence.

Inasmuch as a large-scale condition of homogeneity does exist, there is an analogous condition for an addition of gravitational potentials theorem. How, then, can general relativity be modified to comply?

Suppose the gravitational potential instead of the gravitational escape speed is the primary condition of an upper limit. Unique is (½)c, justified by the slow light condition of Schwartzschild Metric in the manner

$$c\left(1 - \frac{2GM}{Rc^2}\right) = c\left(1 - \frac{1}{2}\right) = \frac{1}{2}c = c'$$

The difference between an escape speed and square root of the gravitational potential is that the former is greater than the latter by the square root of 2. One-half squared is one-fourth. One-half c for the gravitational potential in the relativistic factor thus relates in the manner

$$\sqrt{1 - \frac{2GM}{Rc^2}} = \sqrt{1 - \frac{1}{2}} = \sqrt{\frac{1}{2}}$$

Since the square root of the gravitational potential is the square root of one-half that of the escape speed, its value per relativistic factor equals c, as for an increase of the square root of the gravitational potential to equate as the upper limit for the escape speed as the escape speed c.

There is thus an analogous condition of SRT and GRT regarding a nullification of spacetime curvature due to a homogeneous distribution of mass. Otherwise, light and mass do not move along the same path because of the inhomogeneous nature of gravity. For instance, objects that move at different speeds do not move the same path perpendicular to the direction at which they gravitate towards the center of mass. If you throw a ball faster along a smooth part of Earth's curved surface, it will move farther before it falls to the ground.

The significance of this analysis in this book is that it applies in a latter chapter, The Relativity of Hubble Cosmology, to the average density of the universe and its large-scale homogeneity in determining different values of the Hubble Constant.

QUANTUM ORIGINS

A key origin of the quantum is blackbody radiation. Formulae were derived to determine a relation between temperature and intensity of radiation that a body of mass can absorb and emit while in equilibrium with the forces of nature. Classical formulae for predicting results of experiment failed until a concept of the quantum was introduced. Understanding this development is with regard to particular relations between heat and light.

Heat and Light

In the year 1800, William Herchel (1738-1822) used a variety of glasses of different color lenses peering through a telescope and observing the sun. By using these different glasses of various color, he discovered light filtered through some of them felt warmer than if filtered through other lenses. He followed up with the implications of this discovery by devising experiments consisting of prisms and thermometers to further discover heat tends to be greater towards the red end of the light spectrum and even beyond into the then unknown infrared part of it invisible to the eye.

A heat-light connection was thus established, but it would be another twenty years before Andre-Marie Ampere (1775-1836) suggested light and heat are only different aspects of the same process. A response to Ampere's suggestion came with Marcedonio Melloni (1798-1854) agreeing inasmuch as he believed both are waves propagating through media. He regarded light as the harmonious waves of æther, and he similarly regarded heat as radiant waves of caloric, but his experimental findings from 1833 to 1840 indicated no essential difference in wave properties of these two phenomena. To his credit, he did discover the refractive properties of thermal radiation.

More development followed from experiments by Jean Bernard Leon Foucault (1819-1868) and Armond Hippolyte Louis Fizeau (1819-1896) in confirming the wave properties of radiant heat. Their experiments split rays of infrared light for them to superimpose and produce alternating bands of hot and cold in analogy
to the light and dark fringes of ordinary light. James David Forbes (1809-1861) then discovered heat polarizes similar to that of light, and he advanced the concept of a continuous radiation spectrum that later became essential to Maxwell's theory of electromagnetism. It followed that the unification of electromagnetism and thermodynamics was in order. It would have been routine except for the predominance of wave theory at the time. The relation between vibrant molecules of matter with the waves of light was not adequately understood as of yet.

There had been some attempts to unify theory as such. Leonard Euler (1707-1783) proposed the principle a particular substance of mass can absorb light of any frequency that its smallest particle is able to vibrate. He attempted to explain phenomena according to the æther from which matter forms. His theory was not successful, but his absorbing principle did not go unnoticed.

More essential to the development of theory were discoveries of how matter absorbs light. William Wollaston (1766-1825) discovered in 1802, for instance, that light spectra emitted from matter include dark lines. Joseph Fraunhofer (1787-1826) made more discoveries along this line in 1814. The interest in these discoveries grew among theorists. Stokes, for one, used the principle of Euler to explain them as atoms absorbing light waves by means of resonance.

Pierre Prevost (1751-1839) had provided evidence indicating all bodies radiate heat. The evidence suggested further that poor absorbers of heat are also poor emitters of it, whereas the good absorbers are good emitters, and it became evident materials in thermal equilibrium emits what they absorb. In 1858, Balfour Stewart stated the law that the absorptivity of a material in a state of thermal equilibrium is equal to its emissivity. Stewart assumed the absorptive and emissive ability of different materials varies in relation to the nature of their internal substance. Gustav Kirchhoff (1824-1887) proposed in 1859 a particular condition of "blackbody" radiation applies to all bodies regardless of their material composition.

Kirchhoff examined the spectrum of sunlight after it passed through a sodium flame to discover dark lines of the spectrum change to yellow when the sunlight is of low intensity, being darker with more intense sunlight. He also found sodium emits the same part of the light spectrum absorbed with an appropriate increase in temperature of the sodium flame. Kirchhoff thus surmised the ability of substance to absorb and emit a certain color of light depends on its relative state of equilibrium. Further experiments to confirm this premise indicated absorptivity and emissivity for a material is a function of its temperature and the wavelength, or frequency, of the light alone.

The mathematical formulation of the law assumes a system obtains the state of thermal equilibrium at sub temperature below incandescence. Thus, if A denotes the total radiation per surface area on each body of mass, *a* the fractional amount of radiation actually absorbed by its material, and E is the permissible radiation emitted from it, then the relations are mathematically expressed in the manner

$$aA = E$$
 $A = \frac{E}{a}$

For all materials in a state of thermal equilibrium, the powers of emissivity (E₁, E₂, E₃, etc.) divided by their respective factors of absorptivity (a_1 , a_2 , a_3 , etc.) equal the same amount of light incident per surface area:

$$A = \frac{E_1}{a_1} = \frac{E_2}{a_2} = \frac{E_3}{a_3} \cdots = \frac{E_n}{a_n}$$

Kirchhoff defined the blackbody as one absorbing and emitting radiation of all frequencies or wavelengths, but the power of emissivity of the blackbody is E_B , and the fraction of light absorbed is unity, $a_B = 1$, wherefrom

$$\frac{E}{a} = \frac{E_B}{1}$$

Dividing by E_B , multiplying by *a*, changing order and assuming absorptivity equals emissivity obtains

$$a = \frac{E}{E_B} = \varepsilon$$

The symbols *a* and ε denote absorptivity and emissivity, respectively, of a material body in ratio to a blackbody. Further relations are of the discovery of a fourth power law commonly referred to as the Stefan-Boltzmann Law.

The Stefan-Boltzmann Law

The rate systems change from one temperature to another to obtain a state of equilibrium with its environment is further significant. Newton had assumed the process is linear inasmuch as the rate of change in temperature is proportional to the difference in temperatures between the system and its environment, but experimental data indicated the relation is approximate, as only true at relatively normal temperatures, as the data did not appear linear at higher ones.

Another relation superseded Newton's in the late 19th century. It came from a study of temperature and light intensity by John Tyndall (1820-1893) and Joseph Stefan (1835-1893). Tyndall had run an electric current through a platinum wire that resulted in heating the wire to a state of incandescence. On measuring the radiation emitted at different temperatures, he found the light intensity to be about twelve times greater with a wire being about 1200 degrees centigrade than if only 525 degrees centigrade. Stefan calculated

$$\left[\frac{273^{\circ} + 1200^{\circ}}{273^{\circ} + 525^{\circ}}\right]^{4} = \left[\frac{1473^{\circ}}{798^{\circ}}\right]^{4} \approx 12$$

The 273° is the centigrade scale of 273° above absolute zero. The ratios are thus according to absolute zero.

This mathematical relation appeared the same for all substances at all temperatures. Stefan concluded the intensity I is proportional to the fourth power of the absolute temperature T:

$$I = k_B T^4$$

The constant k_B is named the Boltzmann constant. Its value is 1.3806503 x 10⁻²³ J/K, which is 1.38065 X 10⁻¹⁶ grams multiplied by centimeters squared per seconds squared per one-degree Kelvin. Because of such minuteness of the numerical value of the constant, a slight change in temperature, even to the 4th power, causes only a slight change in light intensity.

Tyndall's measurements were not accurate. Modern results give a ratio of about 18 to 1, but not according to Kirchhoff's black-body condition by which Ludwig Boltzmann (1844-1916) deduced the law in accordance with the second law of thermodynamics.

Boltzmann's derivation includes an exchange of heat between different systems. System S_2 receives heat energy Q_2 from system S_1 . The loss of Q_2 in ratio to absolute temperature T_2 , as negative entropy, equals the amount of positive entropy Q_1/T_1 remaining in S_1 . Along with the exchange of heat are differences in size of energy densities. Energy density u_1 of S_1 is reduced in size from V_1 to V_4 , and energy density u_2 is increased in sized from V_2 to V_3 .

Because the change of states is of heat energy, they interpret in terms of work performed. The work energy by means of an adiabatic process also complies with Boyle's Law whereby a product of pressure p and volume V of a gas at given temperature T is constant: pV = k. Pressure further relates in terms of force F, such as the force of gravity maintaining the pressure in a tire per surface area $2\pi r^2$. With V as proportional to radius cubed, further relations result as

$$pV \propto \frac{FV}{r^2} = \frac{Fr^3}{r^2} = Fr = FV^{\frac{1}{3}} = mc^2r$$

Energy density u relates as mass-energy mc² per volume space. It equates to the amount of force containing it, thus being per volume in the manner

$$u = \frac{F}{V} \qquad F = uV$$

By substitution, the previous equations become

$$uVV^{\frac{1}{3}} = uV^{\frac{4}{3}} = k$$

To convert into terms of volume space, the square-root of the square-root of the equation cubed becomes

$$u^{\frac{3}{4}}V = k^{\frac{3}{4}}$$

Entropy is next considered in relation to different temperatures. States S_1 and S_2 relate as conserved entropy in the manner

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0$$

Q1 and Q2 denote different energy sizes in the manner

$$Q_2 = u_2(V_3 - V_2)$$

 $Q_1 = u_1(V_1 - V_4)$

Hence

$$\frac{Q_2}{T_2} + \frac{Q_1}{T_1} = \frac{u_2(V_3 - V_2)}{T_2} + \frac{u_1(V_1 - V_4)}{T_1} = 0$$

In relation to Boyle's Law, which requires different constants for different temperatures, k_1 and k_2 become the respective constants for T_1 and T_2 , and for different size energy densities in the manner

$$u_{2}^{\frac{3}{4}}V_{3} - u_{2}^{\frac{3}{4}}V_{2} = k_{2}^{\frac{3}{4}} - k_{1}^{\frac{3}{4}} \qquad u_{1}^{\frac{3}{4}}V_{1} - u_{1}^{\frac{3}{4}}V_{4} = k_{1}^{\frac{3}{4}} - k_{2}^{\frac{3}{4}}$$
$$u_{2}^{\frac{3}{4}}(V_{3} - V_{2}) = k_{2}^{\frac{3}{4}} - k_{1}^{\frac{3}{4}} \qquad u_{1}^{\frac{3}{4}}(V_{1} - V_{4}) = k_{1}^{\frac{3}{4}} - k_{2}^{\frac{3}{4}}$$
$$V_{3} - V_{2} = \frac{k_{2}^{\frac{3}{4}} - k_{1}^{\frac{3}{4}}}{u_{2}^{\frac{3}{4}}} \qquad V_{1} - V_{4} = \frac{k_{1}^{\frac{3}{4}} - k_{2}^{\frac{3}{4}}}{u_{1}^{\frac{3}{4}}}$$

Substituting the right-hand sides of these latter equations into the thermodynamic equation obtains

$$\begin{aligned} \frac{Q_2}{T_2} + \frac{Q_1}{T_1} &= \frac{u_2(V_3 - V_2)}{T_2} + \frac{u_1(V_1 - V_4)}{T_1} \\ &= \frac{u_2}{T_2} \left[\frac{k_2^{\frac{3}{4}} - k_1^{\frac{3}{4}}}{u_2^{\frac{3}{4}}} \right] + \frac{u_1}{T_1} \left[\frac{k_1^{\frac{3}{4}} - k_2^{\frac{3}{4}}}{u_1^{\frac{3}{4}}} \right] \\ &= \frac{u_2^{\frac{1}{4}}}{T_2} \left(k_2^{\frac{3}{4}} - k_1^{\frac{3}{4}} \right) + \frac{u_1^{\frac{1}{4}}}{T_1} \left(k_1^{\frac{3}{4}} - k_2^{\frac{3}{4}} \right) \\ &= \frac{u_2^{\frac{1}{4}}}{T_2} \left(k_2^{\frac{3}{4}} - k_1^{\frac{3}{4}} \right) - \frac{u_1^{\frac{1}{4}}}{T_1} \left(k_2^{\frac{3}{4}} - k_1^{\frac{3}{4}} \right) = 0 \end{aligned}$$

Hence

$$\frac{u_2^{\frac{1}{2}}}{T_2} - \frac{u_1^{\frac{1}{4}}}{T_1} = 0 \qquad \qquad \frac{u_2^{\frac{1}{4}}}{T_2} = \frac{u_1^{\frac{1}{4}}}{T_1}$$

Because energy densities and temperatures equal, the adiabatic cyclic change is constant, such that

$$\frac{u^{\frac{1}{4}}}{T} = k^{\frac{1}{4}} \qquad u = kT^4$$

Moreover, u is energy density of the radiation of intensity I absorbed or emitted, as proportional to a change in energy per volume, or as the intensity of a radiation incident on a blackbody. They express different aspects of energy, but they are the result of the same process. By substituting I for u, $I = k_B T^4$ is a mathematical interpretation of the Stefan-Boltzmann's law, with k_B designated as the Boltzmann constant.

For another perspective, eight times energy density for one-half radius needs twice that energy density to change it, whereby $2(8) = 16 = 2^4$.

Wien's Displacement Law

The Stefan-Boltzmann law relates the difference in intensity of radiation to temperature, but it does not specify how either frequency or wavelength of light applies. Wilhelm Wien (1864-1928) determined the relation for the intensity of radiation for a temperature and the range in the wavelength or frequency of the temperature. He noted shorter wavelengths are more frequent, numerous and energetic than longer waves.

For further analyses, Wien adopted the adiabatic concept in relation to blackbody radiation. However, a true blackbody is not available to study within the confines of the laboratory. Although soot is black, for instance, it still emits a radiation invisible to the eye. Even so, blackbody conditions of equilibrium tend to occur with a slight change in the environment from its interaction with radiation. Although an oven allows radiation to escape, the oven temperature can still be maintained by means of using fuel. Wien thus considered a hollow container staying in thermal equilibrium, as by having a tiny hole at its surface to allow radiation to enter and reflect here and there on the walls of the container before it finally finds its way out.

Wien surmised a blackbody varies with a small range in wavelength of radiation at maximum intensity in inverse proportion to temperature. Thus, a shorter more frequent wave at maximum intensity correlates with a higher temperature in the manner $\lambda_{max}T = b$ as Wien's displacement law, where b is a constant.

Relating the intensity of radiation along with both the temperature and wavelength is more complex. Although total intensity is to the fourth power of temperature, determining it according to any particular wavelength at any given temperature needs to include three variables of wavelength, intensity and temperature. Since total intensity is to the fourth power of temperature, since the wavelength shortens per higher temperature, and because of such other considerations as with regard to the Doppler Effect, Wien eventually concluded the intensity for a particular wavelength at a given temperature is to the fifth power of the wavelength.

To derive a formula, he related wavelength and temperature according to a distribution law derived by Maxwell for relating molecular speeds in gas in relation to temperature:

$$dN = \left[\frac{m}{2\pi T}\right]^{\frac{1}{2}} \cdot e^{-\frac{1}{2}mv^2/k_BT}$$

The exponential function $e \approx 2.71828182$ in Maxwell's formula is raised to a power of kinetic energy per k_BT, a product of the Boltzmann constant k_B and temperature T.

Wien associated cavity radiation as molecular resonance and frequency in relation to kinetic energy to derive a function $F(\lambda,T)$ for temperature and range in wavelength λ , with the insertion of constants *a* and b in relation to the intensity I_{λ}:

$$I_{\lambda} \cdot d\lambda = \frac{a}{\lambda^5} \cdot e^{-b/\lambda k_B T}$$

It fared well with data in relation to high frequencies, but not in relation to low ones.

Plank's Solution

Scientists had been aware from the mid-19th century that light escaping from an oven of higher temperature through a tiny crack is more energetic. Therefore, Ferdinand Kurlbaum (1857-1927) and Heinrich Leopold Rubens (1865-1922) experimented in observing waves as long as 59 microns, which is one twentieth of a millimeter. Wien's formula failed in predicting accurate results for these longer wavelengths.

When Rubens revealed the experimental results to Max Planck (1858-1947) in 1900, Planck had a solution the same day. He assumed resonators of radiant heat mediate between molecules and radiation, as for absorbing, storing and releasing the same particular quantity of radiant energy. Planck's solution to the problem was therefore to quantize energy as multiples of hf, wherefore h is now known as Planck's constant, and f is light frequency in relation to energy.

The amount of energy of an oscillator is according to the values of each energy level, which are 0, hf, 2hf, etc. With some values as zero, neither the energy level nor the spatial distribution is continuous. The distribution itself is determined according to an exponential function of the Boltzmann factor

$$e^{-E/k_BT}$$

As to further develop Maxwell's statistical treatment of the kinetic theory of gases, Boltzmann had derived the probabilities in terms of the exponential function, and he also applied discrete energy levels as infinitesimal divisions among the actions of molecules. Planck instead assumed non-infinitesimal-discrete-energy-levels apply.

A number N of oscillators in an incremental range of frequency near f has various multiples of energy E = hf, including zero, and is the number n times each consecutive level of the probability distribution, as in accordance with

$$N = n + ne^{-hf/k_BT} + ne^{-2hf/k_BT} + \dots$$
$$= n(1 + e^{-hf/k_BT} + ne^{-2hf/k_BT} + \dots)$$

The infinite series in the equation converges to

$$V = n \left(e^{-hf/k_B T} \right)^{-1}$$

The total energy of all oscillators is

$$E = n(0) + n(hf)^{-hf/k_BT} + \dots$$
$$= nhf e^{-hf/k_BT} \left[1 + 2(e^{-hf/k_BT}) + 3(e^{-hf/k_BT})^2 + \dots \right]$$

This infinite series converges to

$$E = nhfe^{-hf/k_BT} \left(1 - e^{-hf/k_BT}\right)^{-2}$$

Dividing E by N obtains

$$\frac{E}{N} = \frac{nhfe^{-hf/k_BT} (1 - e^{-hf/k_BT})^{-2}}{n(1 - e^{-hf/k_BT})^{-1}} = \frac{hfe^{-hf/k_BT}}{1 - e^{-hf/k_BT}} = \frac{hf}{e^{-hf/k_BT} - 1}$$

E/N represents the average energy per oscillator of incremental frequency range between f and (f + df) inasmuch as n factors out of the equation.

Comparing Formulas In comparison to Wein's formula, Planck derived

$$I_{\lambda} \cdot d\lambda = \frac{2c}{\lambda^5} \cdot \frac{h}{e^{hc/\lambda k_B T} - 1}$$

A main difference in it and Wein's formula is Wein's constants *a* and b are replaced with 2hc and a different form of the exponential function. As for 2hc, it is twice a change in speed c with regard to a reverse in direction.

The constant h relates dimensionally as a product of mass, velocity and distance, whereas the constant k_B is the product mass and velocity squared. Thus, hc is dimensionally the same as $\lambda k_B T$, such that they are of the same ratio no matter what are units of mass, distance and time used to determine results. The exponential function thus changes only by the ratio of T, which is consistent with Wien's displacement law.

If $c = \lambda f$, then replacing each λ with c/f obtains

$$I_f \cdot df = \frac{2f^5}{c^4} \cdot \frac{h}{e^{hf/k_BT} - 1}$$

However, textbooks give the frequency formula as

$$I_f \cdot df = \frac{2f^3}{c^3} \cdot \frac{h}{e^{hf/k_BT} - 1}$$

The result of using different formula for λ and f is that $c \neq \lambda f$.

The reason the results of these formulas do not equate is because they pertain to different forms of energy. One form is a modification of Wien's formula, and the other is from the formula derived by Sir James Jean (1877-1946) and John William Strutt (1842-1919), renamed Lord Rayleigh. Wien's distribution formula is in accordance with his displacement law for a change in temperature; the other formula was derived according to an equipartition theorem derived independently first by John James Waterston (1811-1883) and later by Maxwell and Clausius for advancing the kinetic theory of gases. It relates to degrees of freedom.

A degree of freedom relates in accordance with the number of modes of vibrations. The same space is assumed to be more capable of containing a greater number of modes of shorter wavelengths. Rayleigh assumed there is tendency for the shorter modes to dominate. However, the shorter waves are more energetic because of their more rapid vibrations, such as to result in an ultraviolet catastrophe from a tendency to increase to infinite energy. Such a result was considered in violation of conservation of energy, and the formula differs from the Stefan-Boltzmann fourth power law. The formula was thus in need of modification.

In Planck's formula, change in wavelength λ relates in the manner

$$\frac{hc}{\lambda^5} = \frac{hf\lambda}{\lambda^5} = \frac{hf}{\lambda^4} = \frac{mc^2}{\lambda^4}$$

The change in radiant energy is thus per fourth power of wavelength. In the other formula, change in frequency f relates in the manner

$$\frac{hf^3}{c^3} = \frac{mc^2f^2}{c^3} = \frac{mc^2f^2}{c(\lambda^2f^2)} = \frac{mc}{\lambda^2}$$

There is thus change in momentum occurring per wavelength squared, as in contrast of internal energy of mass per wavelength to the fourth power.

QUANTUM PHYSICS

An early application of the quantum was the Bohr Theory of the atom. The discoveries of a photoelectric effect and Compton Effect followed. Further advance came with the development of Quantum Wave Mechanics, which is a relativistic quantum modification of Wave Mechanics whereby its wave equations, in turn, became reinterpreted as probability equations according to an Uncertainty Principle. Such new interpretation is now established with the evolution of Quantum Electrodynamics (QED), the quark model, string theory, Grand Unified Theories and so on. Although such development is complex, quantum theory itself can still be explained according to the simpler theory of the Bohr atom in establishing the fundamental relations of physical constants. It provides a basis for understanding a more complex develop for the unification of relativity and QED.

The Bohr Atom

Applying quanta to the structural nature of the atom came from Niehs Henrik David Bohr (1885-1962). He modified the atomic model previously announced by Ernst Rutherford (1871-1937) in 1911, who had contrived a model of the atom to describe how alpha particles reflect. Accordingly, the bulk of mass that scatters is contained within a nuclear radius of about 1836 times smaller than the radius of the atom itself. Rutherford further assumed atoms continually absorb and emit radiation as electrons accelerate around the nucleus. However, his model failed to predict results of all phenomena.

One such phenomenon pertains to the spectra of radiation emitted by atoms. The spectral lines of light associated with atoms did not conform to a pattern consistent with a known theory of continuous spectra. Formulae had been provided apart from theory, as in 1885 by Johanne Jakob Balmer (1825-1890), and the later experiments by Johannes Robert Rydberg (1854-1919), Carl Runge (1856-1927) and Henrich Kayser (1853-1910). Their *ad hoc* formulae, for the most part, agreed with observations, but they lacked the theoretical foundation until Bohr proposed, in 1913, a quantum restriction for his modified version of Rutherford's atomic model.

Bohr assumed electrons orbit the nucleus of an atom in elliptical paths whereby atoms either emit or absorb radiation only when the electron state of angular momentum changes in multiples of $h/2\pi$.

Bohr only modified classical formula by applying quantum restrictions, as in assuming a force field exists consistent with Coulomb's inverse-square-law for electrostatics and magnetism. Electrons thus have a negative unit of charge –e and nuclei have a positive unit of charge e. The force between them is the product of charge per distance r squared:

$$F = \frac{e}{r} \cdot \frac{-e}{r} = \frac{-e^2}{r^2}$$

In comparison, Newton's second law of motion for centripetal force gives

$$F = \frac{-mv^2}{r}$$

Equating the right sides of the last two equations and dividing them in half obtains

$$\frac{-e^2}{2r^2} = \frac{-\frac{1}{2}mv^2}{r}$$
$$K = \frac{e^2}{2r} = \frac{1}{2}mv^2$$

It thus relates to classical formula for kinetic energy K of the electron.

The potential energy T of the electron, as within a conservative field of force, is

$$T = \frac{-e^2}{r}$$

The total energy W is therefore

$$W = K + T = \frac{e^2}{2r} - \frac{e^2}{r} = \frac{-e^2}{2r} = -\frac{1}{2}mv^2$$

The last two equalities give

$$\frac{-e^2}{2r} = -\frac{1}{2}mv^2$$
 $r = \frac{e^2}{mv^2}$

The orbital radius of the electron is the unit of charge squared per twice the kinetic energy of the electron.

Bohr assumed the total energy W becomes zero as r becomes infinite, or whenever the atom becomes ionized because of it losing an electron. By applying quantum restrictions to the atomic radius in using the relations $n\hbar = nh/2\pi = n(mvr)$ and $e^2 = mv^2r$, he deduced

$$r = \frac{e^2}{mv^2} = \frac{e^2 n^2 \hbar^2}{m^3 v^4 r^2} = \frac{e^2 n^2 \hbar^2}{me^4} = \frac{n^2 \hbar^2}{me^2}$$

The letter m represents the mass of the electron, \hbar is the Planck constant h divided by 2π , and n is an integer from one to infinity.

The respective velocity of the electron around the nucleus is

$$v^{2} = \frac{e^{2}}{mr} = \left[\frac{e^{2}}{m}\right] \cdot \left[\frac{me^{2}}{n^{2}\hbar^{2}}\right] = \left[\frac{e^{2}}{n\hbar}\right]^{2}$$
$$v = \frac{e^{2}}{n\hbar}$$
$$\frac{v}{c} = \frac{e^{2}}{n\hbar c} = \frac{\delta}{n}$$
$$\delta = \frac{e^{2}}{\hbar c} \approx \frac{1}{137}$$

The fraction (1/137) is the fine structure constant here denoted as δ .

Bohr further recalculated the energy to eliminate r from the equations. Previous equations give

$$E = -\frac{1}{2}mv^2 \qquad v^2 = \frac{e^4}{n^2\hbar^2}$$

Multiplying both the numerator and denominator by c^2 , and substituting δ^2 for $e^4/\hbar^2c^2,$ obtains

$$E = \frac{1}{2} \frac{me^4}{n^2 \hbar^2} = \left[\frac{\frac{1}{2}mc^2}{n^2}\right] \cdot \left[\frac{e^4}{\hbar^2 c^2}\right] = \frac{-\delta^2\left(\frac{1}{2}mc^2\right)}{n^2}$$

The highest energy level is n = 1 electron, such that

$$E = -\delta^2 \left(\frac{1}{2}mc^2\right)$$

It is the kinetic energy (1/2)mv² minus potential energy mv² of m.

Bohr's next assumption was the atom is able to obtain a lesser energy level by emitting a photon of energy E - E' as a difference of energy levels. This energy, as for a general formula predicting light spectra emitted from atoms, relates to the frequency f of the photon by the equation

$$E - E' = \hbar f = \frac{1}{2}\delta(mc^2)\left[\frac{1}{n^2} - \frac{1}{n'^2}\right]$$

A stipulation applies here for n = 1, and for n' to only be any integer greater than one, as for an atom to either decrease or increase from an energy level to another energy level by either absorbing or emitting a photon of energy hf wherefrom the frequency of light is less than the frequency of m.

The Photoelectric Effect

Planck interpreted the nature of the quantum as a molecular oscillator, but Einstein interpreted it as applying to light as well as to the oscillators in explaining the photoelectric effect that was discovered in 1902 by Phillepps Lenard (1862-1947). Lenard discovered the result of electrons emitted from a metal due to light shining on the metal depends more on the frequency of light instead of its intensity. No electrons are emitted if the frequency is too low, and only the number of emitted electrons depends on light intensity.

Einstein interpreted Lenard's findings according to Planck's quantum condition of energy as discrete multiple units of hf. Light quanta now called photons therefore collide with a metal such that the energy of each photon transfers to an electron for the electron to break loose its bondage with the metal. The energy E of the particle of light is called a photon.

Even though the electron only absorbs a particular light quantum, the possible energy of light is still continuous along with energy of matter with regard to relative motion. The continuousness of light energy is associated with varying wavelength λ and varying frequency f with regard to how they interact with matter according to the Doppler principle of relative motion. The quantum relation h/ increased by a relativistic factor for relative motion of matter, for instance, allows continuous energy of light to correspond to continuous energy of matter in relative motion such that more intense light reflected by matter can increase thermal temperature and kinetic energy, but the ejection of electrons is still determined by the quantum states of matter and light, as too frequent of a collision for elastic response of a metal.

After determining a numerical value for the fundamental charge of the electron, Robert Andrews Millikan (1868-1953) was able to verify Einstein's explanation of the photoelectric effect as well. His experiment was that of a statistical nature in measuring the electrical voltage of electron emissions in comparison to the light intensity on a metal screen. It produced a photon to electron emission in accordance with the light energy needed of a particular frequency. In 1915, Millikan's controlled experiments were able to interpret the data in a way it not only convincingly verified the photoelectric effect, it confirmed the value of Planck's constant as well.

The Compton Effect

Similar to the photoelectric effect is the Compton Effect that includes the redirection and loss of photon energy as well as an ejection of electrons from atoms. However, only reflection between light and free electrons apart from containment, such as by a metal, are observed. In explaining statistical results of electron and light interaction, Arthur Holly Compton (1892-1962) assumed light and electrons are particle-like. Thus, the laws of conservation of energy and momentum apply to statistical results for equating with a loss of energy of photons, as according to their angle of deflection, and according to the gain in the energy of electrons recoiling in opposite directions.

In 1922, Compton bombarded graphite with x-rays in examining how x-rays scatter electrons. He discovered their angle of scattering is consistent with conservation of momentum. The momentum of a scattered electron at a particular angle of a scattered electron coincides with a loss in momentum of the scattering x-ray and its recoil angle, as according to a Doppler shift to a longer wavelength. In manner of conservation of momentum, a change in the wavelength of an x-ray after it collides with an electron is

$$\Delta \lambda = \lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

With straight ahead collision the cosine angle is unity such that 1 - 1 = 0, as zero change in wavelength. Since the right-angle reflection is a zero-degree cosine angle, the Compton wavelength for the electron is

$$\lambda_C = \frac{h}{m_e c} = 2.43 \times 10^{-10}$$
 centimeters

For a 180-degree reflection, the cosine angle is minus unity, as for a change in wavelength that is twice the Compton wavelength. The Compton wavelength λ and frequency *f* represent properties of a photon moving at speed c. For interacting with an atomic particle, they also relate to a quantum h of energy and mass m relatively at rest in the manner

$$E = hf = \frac{hc}{\lambda} = mc^2$$

Note: Kinetic energy K of the electron around the nucleus of the hydrogen atom is $K = (\frac{1}{2})m_ev^2 = hv/2\pi r_a = \hbar v/r_a$. The fraction K/E is such that v/c equals the fine structure constant of 1/137.036.

Wave Mechanics

William Rowan Hamilton (1805-1865) formulated wave mechanics to combine wave properties with particle interaction. He noticed a similarity in form in mathematical formulations of the principles of least-time and least-action. Pierre de Fermat (1601-1665) offered the principle of least-time as a means to describe a light path. The least-action principle was formulated by Pierre Louis Moreau de Maupertis (1698-1759) for describing the dynamics of interaction between particles.

Properties of various physical media for the propagation of light waves are according to their refractive indices, as defined by the equation

$$\mu = \frac{c}{v}$$

The Greek letter μ denotes the refractive index of the medium, c is the light speed in vacuum space (or æther), and v is light speed through a physical medium. The refractive index for vacuum space or æther is unity. It is greater than unity for other media. Since c is constant, and since $\mu v = c$, the speed of light is slower in a refractive index greater than one. The slower speed of c is justified by conservation of momentum since the wave of increased inertia is of the denser medium. An interpretation could be the denser medium converts light to mass.

Fermat's principle of least time is according to the relation where time along the total path from point A to point B at each increment of distance ds through a medium of refractive index u is

$$\int_{A}^{B} u \cdot ds$$

A particular requirement of this integral is it needs to be an extremum, as either a maximum or a minimum value. The value of least time comes by way of associating u with c/v, such that the greater u results in a slower v for more time. The significance of this integral with regard to the law of refraction is in realizing a total path that light takes through two media of two different indices. Light paths from A to B through the first medium and from B to C through the second medium need to be a minimum for the total time. The least time does not mean the shortest path; it, rather, depends on the angle of entry. The shortest path is a straight line from A to C, but the time along this path is greater for the medium having a greater refractive index. If the angle of entry is from a shorter path of the medium of a greater index, then the time can be less in the new and longer path. The law of refraction also expresses this result. The least time principle is thus according to the law of refraction in view of wave theory.

> A straight line is shorter from point A to point C, but the path from point A to point B to point C is faster because of the shorter distance from point A to point B in the denser medium, M_1 .



The principle of least action is similarly expressed as

$$\int_{A}^{B} mv \cdot ds$$

Momentum mv replaces refractive index μ of the medium. An increment of distance ds along some path from A to B thus has the momentum mv for action. For the total path from A to C through two media of two different indices of reflections, the same condition exists for less action as it does for less time. The least action principle thus determines the path of a particle in a conservative field of force in the same manner the principle of least time determines the path of the light wave. A conservative field of force is one in which the total energy, as potential and kinetic, stays the same.

The Hamiltonian Action is a formulation of Least Action in view of a conservative field of force. A constant force in a homogeneous medium is simply the product of a particle's momentum mv and its distance moved r. Total energy before, during, or after a particle passes, is the potential energy of a field in association with the particle in addition to its kinetic energy of relative motion. For constant field energy, Hamilton distinguished between the action of the particle and total field energy as varying in time and place. The Hamiltonian Action A thus takes the form A = S - Et. E denotes total energy, t is the time of action, and S is the Maupertuisian Action mvr.

Anywhere along the path of the particle, or along the crossing of paths of a swarm of particles, the action varies with time. At any place where the total positive and negative actions as zero are the relations

$$Et = mvr$$
$$\frac{E}{mv} = \frac{r}{t} = v$$

An internal action of a field in equilibrium is thus describable.

Hamilton's Wave Mechanics are significant in describing the evolution of waves as multiple interactions of particles and manifestations of particles in an association with wave packets. A mathematical theory of wave packets was later developed by Lord Rayleigh. It foreshadowed the wave mechanics of de Broglie and Schrodinger that included Planck's constant h.

Quantum Wave Particles

The discovery of the quantum along with the success of wave theory for describing the behavior of light indicates light has a dualistic nature. It behaves partly as a particle and partly as continuous waves. Louis de Broglie (1892-1987) further theorized the nature of matter as a particle-wave duality as well with regard to his formulating theory according to special relativity, wave properties and the quantum. Accordingly, energy E and momentum P of a particle moving freely in space relate to the number σ of waves having frequency f and wavelength λ equate with Planck's constant h in the manner

$$E = mc^2 = \frac{m_o c^2}{\sqrt{1 - \beta^2}} = hf$$
$$\vec{P} = m\vec{v} = \frac{m_0 \vec{v}}{\sqrt{1 - \beta^2}} = \frac{hf}{\vec{V}} = \frac{h}{\vec{\lambda}} = h\vec{\sigma}$$

The arrows indicate the waves and particles move in the same direction.

Capital V denotes the velocity of the waves. As with waves in general, it equates to the product of their wavelength λ and frequency f as

 $V = f\lambda$

This equation in relation to momentum and energy further equates as

$$V = \lambda f = \left[\frac{E}{h}\right] \cdot \left[\frac{h}{P}\right] = \frac{E}{P} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

The case of photons moving at speed c gives

$$V = \frac{c^2}{v = c} = \frac{c^2}{c} = c$$

Since special relativity stipulates no information of events can exceed light speed in vacuum, the de Broglie waves carry no momentum or energy for them not to transmit direct effect to the observable world.

De Broglie depicted an electron orbiting the nucleus of the atom as a superposing of waves creating effects similar to the beats of sound waves of the vibrating strings of a violin.

Wave equations relating musical harmony of musical instruments were developed in the 18th century. Particularly, Jean le Rond d' Alembert (1717-1783) proposed, in 1746, a one-dimensional wave equation for a string-like vibration:

$$\frac{\partial^2 u}{\partial^2 x} = \frac{1}{v^2} \frac{\partial^2 u}{\partial^2 t^2}$$

The letter u represents the amplitude of waves as a function of energy, time and distance: energy being proportional to the square of the amplitude. The symbol ∂ represents partial differential, x its axis of direction in time t, and v its velocity. Although the equation is one dimensional, a plane is indicated by string-like vibration along a y-axis, such that the average displacement is

$$\left(v\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right)\left(v\frac{\partial}{\partial x} - \frac{\partial}{\partial t}\right)y = 0 = \left(v^2\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2}\right)y$$

The equation thus allows for solutions of internal action.

A three-dimensional wave equation was proposed three years later by Leonard Euler (1707-1783) with the symbol ∇ for three perpendicular axes. Erwin Schrodinger (1887-1961) also transformed de Broglie's particle-wave duality into a three-dimensional-space quantum wave mechanics in deriving more wave equa-

tions, such as a non-relativistic one for a particle moving in an electric field of the form

$$i\hbar \frac{\partial}{\partial t} \Psi(r,t) = \left[\frac{\hbar^2}{2u} \nabla^2 + V(r,t)\right] \psi(r,t)$$

 $\boldsymbol{\psi}$ is the Schrodinger symbol for wave function, as for (r,t), ∇ is the Laplace symbol for three dimensional (x,y,z) coordinates, V represents total kinetic and potential energy according to Hamilton's conservation field rule, i is an imaginary number for the square root of -1, u is a reduced mass of a particle, and \hbar is the reduced Planck constant by 2π . The equation is in compliance with conservation of energy according to classical theory.

Schrodinger's wave equation with quantum restrictions reinterprets the quantum conditions of the atom that was previously given by Bohr. Bohr had postulated a correspondence principle for including a wave condition where needed. Bohr's model further describes electrons circling the nucleus of the atom as being corpuscular in nature, whereas Schrodinger considered electrons as a diffused cloud whereby quantum restrictions result as certain nodal points of de Broglie standing waves.

A particular significance of Schrodinger's wave interpretation is Bohr's theory only explains refractive properties of atomic spectra as arising from an electron's change of orbit, as one of more energy closer to the nucleus, whereas refractive index of a medium for wave propagation explains why there is greater energy of attraction closer to the nucleus as a reflection of more 'virtual' light energy.

Probability Interpretation

Leading physicists as Max Born (1882-1970), Werner Karl Heisenberg (1901-1976), and Niehs Bohr reinterpreted Schrodinger's wave equations as probability equations. Born proposed the wave packet associated with the intensity of the action is the probable whereabouts of a particle. Heisenberg and Bohr then associated both waves and particles according to probability.

Born first reinterpreted Schrodinger's theory. What Schrodinger had proposed is a cloud of negative charge in place of Bohr's original model of the particle circling the nucleus of the atom. For instance, in the hydrogen atom, which consists of one electron, as one unit of charge, the total charge of the electron cloud, as Schrodinger had envisioned, is –e. In addition, he proposed the product of charge and an equilibrium state of wave intensities, as the product of amplitudes equals its energy density ρ at any point dxdydz when the energy of the atom is in a stable state:

$$\rho = -e\psi_n\psi_n^*$$

The product of the wave functions is proportional to the wave intensity. It is used instead of a mere wave function squared for it to allow their product to be negative for a positive density-being a negative density in the natural world is incomprehensible. Representation of total effect over infinite space is by integrating pointcharge-density at coordinate point dxdydz. However, total charge of the cloud is still only

$$-e \iiint \psi_n \varphi_n^* dx dy dz = -e$$

 $\iiint \psi_n \psi_n^* dx dy dz = 1$

A proportion of the charged cloud therefore condenses at one moment to a specific and stable location dxdydz.

Heisenberg and Bohr interpreted the condition according to principles of uncertainty and complementarity. They found it difficult in view of the particle-wave paradox to conceive of a single particle obeying interference of two wave patterns after they pass through only one of two holes. They preferred instead to view the particle as a sort of diffused cloud capable of interfering with itself, or as consolidating into a particle-like form in that the nature of particle-wave effect is neither wave nor corpuscular in an ordinary sense; it manifests instead as certain observable effects of whatever it is that causes them to appear as such. A diffused cloud represents the effect when less observed; a tinier, more precise wave packet represents the effect when observed more accurately.

Although Schrodinger depicted the diffused cloud of negative charge as well, the depiction by Heisenberg and Bohr differed as according to their view of an electron or any other particle inasmuch as they interpreted wave equations as probability equations of how particle effects can be observed.

An explanation of the probability condition could be it is because total energy moving through space is greater than what is actually observed. The observable secondary effects of nature, as Gassendi had proposed centuries earlier, could arise from the primary unobservable source underlying nature. However, without means of verification, the explanation is not science; it is speculation. Nonetheless, by the probability conditions of quantum physics suggesting an existence of a virtual field of virtual particles, the philosophy of Gassendi is worthy of consideration for a more complete understanding regarding wave-particle duality and physics in general.

Uncertainty

Heisenberg established a principle of uncertainty in order to determine the probability of finding the location or time of a particle effect from the probable outcome of its momentum or energy, respectively. According to Born's interpretation of Schrodinger's equations, a wave-packet defines the region where a particle is locatable. The probability of the particle's position is at any point within the wavepacket where it is proportional to the product of the total volume of the wave packet and its relative intensity. The relative size of the wave packet is dependent on observation, as by the ability of a photon to penetrate a certain level within the wave packet. Photons having higher frequency, greater momentum and more energy penetrate deeper into the wave packet for a more precise location of the particle. Photons with less frequency determine instead more exact momentum from the packet's larger size.

Heisenberg's uncertainty principle follows from how Born viewed the wave packet. By Born, any one of the waves in the packet is representative of a probable particle of particular momentum and energy according to the range in energy by which waves vary. The causes of their spreading apart by impact of the photon, and by the difference in energies of waves, render the momentum and energy more uncertain. In following this lead, Heisenberg surmised more exact determination of the particle's position or time by the more energetic photon causes more uncertainty of the particle's momentum and energy. Conversely, the determination of momentum and energy by the less energetic photon leaves more uncertainty in position.

The relative momentum of the photon in relation to wave parameters and Planck's constant h is provided in relation to the equation $P = \hbar/\lambda$. The uncertainty of the change in momentum ΔP with regard to the wavelength of the photon is opposite to the uncertainty of its change in position Δx . Certainty of position is uncertainty of momentum; conversely, certainty of momentum is uncertainty of position. Therefore, a range of positions for each momentum and a range in momenta for each position exist. The total uncertainty is, in wave present form, the product

$(\Delta x)(\Delta P) \ge \lambda(\hbar/2\lambda) = \hbar/2$

Total uncertainty further includes energy and time. The dimensions of h (as mass-velocity-distance or mass-distance squared per time) can also be interpreted as the product of distance and momentum (mass multiplied by distance and velocity) or as the product of time and energy (time multiplied by mass and velocity squared):

$(\Delta t)(E) \ge \hbar/2$

This probable uncertainty is not interpreted the same as is that of flipping a coin. The coin can come up heads 3 times and tails 7 times after 10 tosses. After a million flips the heads-tails ratio is more apt to approximate as 1. In contrast, the probability of quantum physics is an exact prediction. If there is a probability of a particle showing up 3 times out of 10, then it does so as predicted. It can show up 1 time for the first 8 observations, 1 time for the 9th observation, and 1 time for the 10th observation, or otherwise, but it is predicted to show 3 times in any combination of 10 observations.

A particular result of the Heisenberg Uncertainty principle is a virtual field of virtual particles. Such atomic particles as electrons surrounded by an electromagnetic field interact with it in creating additional particle effects. A particle interaction occurs with electrons emitting and absorbing, or merely reflecting, virtual photons according to a Feynman diagram describing their paths. Energy and momentum, within minute time spans, are not conserved, but positive and negative states of energy eventually balance out. A virtual photon can thus turn into a virtual electronpositron pair and vice versa. However, the virtual particles are short lived and are not directly detected, as they merely affect the measure of physical quantities according to the Heisenberg Uncertainty principle.

A virtual particle of interest is the graviton. Consider the probability of detecting gravitational radiation, as by its interaction with mass, is extremely less probable in allowing a long range of counter action as a vacuum effect. In view of the Planck parameters mvr of \hbar , whereby v/c = 1/137.036 is the fine structure constant, substitute the Schwartzschild radius R as the range whereby changes in speed add up to c. Also substitute H₀r' for v. H₀r' here represents the change in speed in relation to the Hubble Constant H₀ for a distance r', where r' is 137.036 times shorter than r, which is representative of either the atomic or nuclear radius of the hydrogen atom. (The shorter r' relates to quicker light action than mass action per time.) Because H₀R = c = vr/r', H₀r'R = vr, whereby gravitons comply with a probability condition of Quantum Electrodynamics. General relativity and QED thereby equate.

Unity Maintained

Schrodinger's wave mechanics explained nearly all conditions of light spectra that Bohr's theory explained except for one that became explained with the *ad hoc* assumption of spin. The spin appeared needed for explaining a particular spectrum. Bohr's theory explained it with the assumption of the magnetic moment of the electron being defined by its own spin, as defined as half the value of the quantum constant ħ. Nowadays it is simply said that the electron has a spin of ¹/₂, but the spin is symbolic in concept insofar as it does not compare with the spin of a ball.

George E. Uhlumbeck (1900-1988) along with Samuel A. Goudsmit (1902-1978) proposed a spinning electron to correct certain inadequacies of Bohr's theory to describe the finer spectra detected from the hydrogen atom. Although it allows for the spinning electron, there is no reason for it, but Wolfgang Pauli (1900-1958) showed the electron spin is consistent with wave equations presented in the form of a matrix. The matrix allows for the splitting of the wave equation as representative of an electron spin in either of two directions. When combined with the amplitude function ψ of waves, it further suggests polarization.

Charles Galton Darwin (1887-1962) then derived a wave equation that includes a polarization effect of the spinning electron in describing the fine structure of the hydrogen atom by means of a wave equation. Neither Pauli nor Darwin, however, derived wave equations consistent with the theory of relativity. This was a task taken on by Adrien Maurice Dirac (1902-1984) in deriving a relativistic form of Schrodinger's wave equations.

Schrodinger succeeded in deriving wave equations in classical form, as consistent with conservation of potential and kinetic energies:

$$\frac{1}{2m} \left(P_x^2 + P_y^2 + P_z^2 \right) - K = 0$$

For wave interpretation, Schrodinger replaced momentum-vectors P_x , P_y , P_z with respective operators for wave interpretation of particle effects:

$$\frac{h}{2\pi i}\frac{\partial}{\partial x} , \frac{h}{2\pi i}\frac{\partial}{\partial y} , \frac{h}{2\pi i}\frac{\partial}{\partial z}$$

In addition, he replaced the energy K with the operator

The operators above are only in compliance with conservation laws of classical mechanics, not those of relativity theory. Schrodinger attempted a relativistic version, but he was unsatisfied with the result. Another one was proposed by Walter Gordon (1893-1939) and Oskar Klein (1894-1977), but it also had reservations with regard to determining the probability density of a particular state of particle motion. For covariance, the relativistic form of the wave equations requires symmetry, which was complicated because the Schrodinger equations in wave form indicated possible negative densities of matter, which was difficult to accept by physicists at the time.

Negative values of relativistic invariance are still possible, but the form of the energy equation requires a more complex solution for including SRT conditions of combining energy with momentum according to the invariant

$$E^2 = P^2 c^2 + m_o^2 c^4$$

In the form of three dimensional coordinates, the invariant is

$$P_x^2 + P_y^2 + P_z^2 + m_o^2 c^2 = \frac{E^2}{c^2}$$

The significance of this invariance is that it is according to four dimensional spacetime in connecting to light speed and electromagnetism.

Including relativistic invariance in wave equations can unify theory, but the unification does not mean one theory becomes the same as the other; it means each become part of the other as parts of a more embracing theory. As the result, the new theory describes more phenomena, as exemplified by Dirac's inclusion of relativistic invariance in quantum wave mechanics.

Invariance of relativity allows a calculation of unknown variables from known ones. Significant with regard to the derivation of the invariance is a nullification of factors that cancel each other out. Derivation of Minkowski invariance does, in fact, consist of nullifications by way of cancellation: The Lorentz transformations for coordinates of time t' and distance s' relate as ct' and s'; the transformation equation of s' is both subtracted and added to the ct' transformation equation; the results of added and subtracted values are then multiplied whereby the resultant value is the same for ct - s. Wave theory connects in a sense to this nullification in that super-imposing waves of opposite phase cancel each other out in effect.

Wave interpretation provides insight into the physical natures of such nullifications, which entails a different interpretation of the mathematics. A different commutative rule for multiplication, for instance, was applied by Dirac for the application of his matrices in allowing Schrodinger's quantum wave mechanics to connect with special relativity theory. The commutative rule means A + B = B + A, as does AB = BA. To the contrary, $A - B \neq B - A$ is non-commutative, as 5 - 3 = 2is positive and 3 - 5 = -2 is negative. Generally, $3 \ge 5 \le 3 = 15$ is commutative, but not according to Dirac's matrix rules.

Of particular significance to the non-commuting rule is its compliance with the number i as the square root of -1. In ordinary algebra, the number one can either be $(1)^2$ or $(-1)^2$. The symmetry of Dirac's matrices, however, is such that $i^2 = -1$ has a unique interpretation: For every positive solution there is a negative or an imaginary solution as well. In reference to particle effects of electrons, for instance, the positive and negative factors represent opposite directions of counterclockwise or clockwise electron spin, and for antimatter to exist as an opposite state of matter.

Further significance is with regard to the Pythagorean Theorem, which is a foundation of relativity theory. For instance, consider

$$(A + B)^2 = A^2 + AB + BA + B^2 = A^2 + 2AB + B^2$$

AB + BA = 2AB does not apply to Dirac matrices. Instead, BA is negative if BA is positive, and A and B represent coefficients whereby AB + BA = 0 and $A^2 = B^2 = 1$ for light speed as unity. The non-commuting rule is thus a means of interpreting

the positive and negative subtractions for a derivation of Minkowski invariance from the Lorentz transformations.

Further significance is with regard to dimensionality. The matrices can increase to include four dimensional spacetime in compliance with relativity theory, but the particular significance of the non-commuting rule along with the number i for positive and negative values is what provides insight into a symmetry of relativistic covariance with regard to spin and antimatter.

This indication of antimatter is a square root of the momentum-energy equation being either positive or negative. It need not necessarily be either, as positive numbers each squared and added have only a positive value for the square root of their summation, but Dirac's matrices are of a symmetry consistent with the interpretation of Schrodinger's equations by Bohr, Born and Heisenberg whereby the wave aspect merely represents the probability of existence, whereby Dirac's theory is verifiable as a test of predicting the positive and negative symmetry of nature.

An even greater significance of Dirac's approach is with regard to the unification of Quantum Mechanics and Relativity Theory. This unification is in relation to spin. Bohr had accepted a suggestion to assume it applies as another condition of the electron in its orbit about the nucleus, as with a ¹/₂ spin of the electron to account for the empirical findings of a Stern-Gerlach experiment. It was discovered that a beam of atoms splits into a number of parts depending on the angular momentum of the atoms, even for angular momentum small as unity. This result indicated atoms split into three parts: -1, 0, 1, such that splitting the beam resulted in three parts, being positive, neutral and negative for half angular momentum in one direction balancing half angular momentum in the opposite direction.

Wolfgang Pauli (1900-1958) proposed in 1924 a new quantum degree of freedom with two possible values to explain observable effects of nuclear spectral. In 1926, he offered a 2 x 2 matrix for describing the two degrees of freedom in manner of non-relativistic spin of the electron. Shortly later, but independently, Dirac revealed a 4 x 4 matrix as a relativistic version of spin, as in accordance with the Minkowski invariant for energy and momentum. Dirac thus succeeded in uniting Quantum Mechanics with special relativity, but not with general relativity in view of the singularity.

Previously proposed in this book is the number ½ as unique to general relativity relating gravitational potential to escape speed squared in avoiding the condition of the singularity and suggesting it could be instrumental for unifying general relativity as well. It relates directly to the ratio of the escape speed-squared to gravitational potential with regard to angular momentum as orbital speed-squared.

Similarly, a significant factor of a Fourier transform is the square root of $\frac{1}{2}$. Joseph Fourier (1768-1830) is noted for his mathematical analyses of heat transfer and his awareness of the greenhouse effect, but application of his mathematics applies to other areas of science as well. The square root of $\frac{1}{2}$, for instance, applies to the amplitude A of a plane wave superposition in the manner

$$A(k) = \frac{1}{\sqrt{2}}e^{-(k-k_0)^2}$$

It relates to a wave-packet moving to the right and left according to a wave function w(k) = kv whereby u(x,t) equals both $F(x \pm vt)$ and further relates to localized disturbance of superimposed waves formed into a wave packet. Particular amounts of wave frequencies are needed to sustain a wave packet whereas destruction of any possible wave packet from it occurs instead. The result is consistent with Huygens' Principle of every point of disturbance is the creation of more waves. Instead of waves being annihilated behind the front progression of wave action, they disperse from being a wave-packet.

The main reason here for pointing out the significance of the fraction $\frac{1}{2}$ is for relating quantum wave mechanics to general relativity. The spin $\frac{1}{2}$ of quantum mechanics relates as angular momentum. Similarly, gravitational potential, as in relation to orbital speed squared, is $\frac{1}{2}$ a gravitational escape speed at the same distance from the center of mass. If the limiting aspect of gravity, as a Schwartzschild radius for an escape speed equal to that of light, as actually conditional to the gravitational potential instead, then the escape speed of $\frac{1}{2}$ would be perceived by an observer at a gravitational potential of $\frac{1}{2}$ as c:

$$\frac{\frac{1}{2}c}{\left(1 - \frac{2GM}{Rc^2}\right)} = \frac{\frac{1}{2}c}{\left(1 - \frac{1}{2}\right)} = c$$
$$\frac{c\sqrt{\frac{1}{2}}}{\sqrt{1 - \frac{2GM}{Rc^2}}} = \frac{c\sqrt{\frac{1}{2}}}{\sqrt{1 - \frac{1}{2}}} = c$$

Gravitational potential can thus be relatively perceived as escape speed.

THE RELATIVITY OF HUBBLE COSMOLOGY

Why an infinite number of stars do not light up the sky at night is known as Olbers' Paradox. Aristotle had proposed a finite number of stars. Kepler had also considered the night sky indicates a finite universe. A century later, Edmun Halley (1656-1742) argued, to the contrary, an uneven distribution of stars could allow for the dark sky. In 1743, Jean Phillippe Loys de Cheseaus (1718-1751) considered the universe is finite unless space somehow absorbs starlight (as does Tired Light theory). The debate continued. Heinrich Wilhelm Mauthaus Olbers (1768-1840) considered an interstellar dust as the absorbing medium of space can allow for an infinite universe of infinite light.

A main criticism of the interstellar dust theory is the belief that dust in thermal equilibrium absorbs enough light from infinite stars to heat up and reemit the light. However, Eddington and others showed that if the overall temperature of the universe is only about 3° Kelvin, then there can still be a state of thermal equilibrium for some other process than that of just heating of the medium. According to Tired Light, for instance, space partially absorbs light energy. However, such theory is not presently established.

Presently established is Big Bang theory assuming the universe is finite and expanding. Tired Light has been dismissed for several reasons. For one reason, as according to Big Bang proponents, how collisions of light with dust particles do not result in a distortion of the cosmic background is in need of explanation. Another reason that has been given for the dismissal of Tired Light has been a Tolman Brightness Test supposedly distinguishing between redshift effects in favor of recessional speed instead of absorption of light energy by the medium of space through which it propagates. However, the Tolman Brightness Test in favor of Big Bang is not supported by astronomical data.

Big Bang

A birth nature of the universe in the form of crystallized matter from an explosion of energy was proposed by Robert Grosseteste (1175-1253) as early as 1225. The more modern version of the cosmic explosion began in the 1910s when Vesto Slipher (1875-1969) began observing radial velocities of galaxies. Carl Wilhelm Wirtz (1876-1939) observed there is more redshift in the light spectrum from more distant sources. In 1929, Edwin P. Hubble (1889-1953) determined a systematic redshift-distance relationship. He and Milton Humason (1891-1972) then formulated the Hubble Law with regard to redshift per distance of cosmic light sources. A plausible explanation of it is the Doppler principle of light spectrum affected by the relative motion of its sources, as because of the galaxies of stars receding from us as well as from each other.

Einstein had inserted a Cosmological Constant into his field equations as a repulsive force counter to gravity in assuming the universe is finite and static, but an analysis of the field equations by Alexander Friedman (1888-1925) indicated the finite universe is not static even with the Cosmological Constant. Solutions to Einstein's field equations were then derived with the intent of explaining an expanding universe. Georges Lemaitre (1894-1966) had suggested in 1927 an expanding universe is traceable back to a time of origin. What developed is the now standard FLRW Metric that is named in honor of Friedman and Lemaitre along with contributors Arthur Geoffrey Walker (1909-2001) and Howard Percey Robertson (1903-1961).

The FLRW Metric is conceptually as well as mathematically complex. It includes a Cosmological Principle with regard to an observable expansion of the universe being conditional to homogeneity and isotropy. Conditional to it, for instance, is observers not actually located at the center of the universe still perceive themselves as relatively located at or near it, as consistent with special relativity whereby any system of inertial motion can be considered to be relatively at rest. Faster recessional speed is thus relative.

By the principle of equivalence of general relativity, covariance also applies to a homogeneous condition of a gravitational field. Light paths are thus curved whereby an observer at the edge of the universe perceives it as near its center. As to how this curvature maintains while the universe expands is more complex. As the universe expands, spacetime curvature of gravity decreases. However, as the universe expands, less spacetime curvature need not be needed.

The origin of the Big Bang is assumed a singularity expanding about 13.7 billion years ago from a dimensionless point space. The expansion rate inflated, within an extremely minute part of a second, faster than the speed of light until becoming comparable to a golf ball. Its rate of expansion then decreased, cooling, to allow mass to form. After a second, neutrons formed to suddenly fill the universe. Further expansion of the universe has cooled it to its present average temperature of about 2.7 degrees Kelvin, as assumed, observed and calculated in the 1960s. The universe initially was supposedly too hot for even light to shine. It did not shine until about 380,000 years to become a microwave background radiation of today with no common origin of location. The discovery of this radiation in the 1960s was what decided it in favor of Steady State, which is another theory of expansion whereby more stars and galaxies are created as other stars burn out of existence as the universe expands.

A timeline is an essential aspect of Big Bang. Simple explanation of it is twice distance equals twice the rate of recession. Assume a value of the Hubble Constant is 70 (km/sec)/Mps. Dividing 70 km/sec by a million parsecs (30.9 million kilometers) equals 2.27 X 10⁻¹⁸ of something per time. If the something is one centimeter, then the result is centimeter per second. If c is an upper limit for a Hubble distance R_p of the universe, then $R_p = c/H_0 \approx 1.32 \times 10^{28}$ centimeters. Dividing R_p by c obtains a time of about 4.4 X 10¹⁷ sec $\approx 7.3 \times 10^{15}$ min $\approx 1.2 \times 10^{14}$ hr $\approx 5 \times 10^{12}$ days $\approx 1.4 \times 10^{10}$ as 14 billion years. However, although R_p represents the present distance of the light source, it is not the distance where it is seen at the present.

Another formula for distance d and present time T_p , as about 14 billion years, is d/v + d/c = T wherefrom d/c is the time for light to reach Earth from its source after receding distance d at speed v during time d/v. The age of the universe thus maintains even though the actual distance observed is not the same distance when light was emitted from its source.

The current position of a light source being calculated from where it is seen in the past is in relation to the Tolman Brightness Test that was used to discard Tired Light in favor of Big Bang. For a light-source-speed of (½)c, it moves one-half the distance light does to reach the observer. The emitted-distance is thus two-thirds the present-distance.

This progression into the past relates to the Tolman Brightness Test in relation to light intensity decreasing per distance-squared while spreading to a larger surface area of an imaginary sphere from its source. Because of the source appearing closer than it actually is, light intensity of the past appears more intense. In contrast to it is Tired Light whereby the Hubble Constant is explained as a gradual decrease in light-energy while propagating through the medium of space.

By Tired Light, if light-energy decreases in proportion to its energy, then the Hubble Constant at three-half distance is actually the average distance of one, onehalf and one-fourth distance, as eleven-twelfth longer distance as measured by light intensity according to Big Bang. The difference ratio of longer distance to shorter distance is eighteenth-seventh. However, if the absorption of light-energy is by the space medium, as constant, then the redshift is constant as well.

Cosmic Modifications

Modifications are often proposed of theory. For instance, Dirac considered a necessary decrease in value of the Hubble Constant regarding what appeared at the time of some aspects of cosmic expansion being to coincidental in nearly equating to atomic parameters. As the universe expands in adding more distance between galaxies, the Hubble Constant should decrease in value for such equality not to be probable, as the Hubble Constant is only constant to a particular time if it is expanding to increase the distance between light sources. To explain the apparent coincidences, Dirac speculated on a solution in relation to how the Hubble Constant relates to the structural nature of mass. For instance, the gravitational constant G, electron mass m_e, light speed c and Hubble Constant H, as determined at the time within an accuracy of about 100 kilometers per second, seemed to relate in the manner

$$\frac{e^4 H}{Gc^3} \approx m_e^3$$

The Hubble Constant then relates in the manner

$$H \approx \frac{Gm_e^3 c^3}{e^4} = \frac{Gm_e^3 c^3}{m_e^2 c^4 r_e^2} = \frac{Gm_e}{r_e^2 c} \approx 2.55 \times 10^{-20} \, \text{sec}$$

The gravitational force of the electron per light speed thus approximated to the then accepted value of the Hubble Constant. However, with a decrease of H with time, as the universe expands, Dirac believed that another one of the constants needed to decrease along with H. Dirac chose G.

A decrease in gravity as the universe expands in size further relates to the decrease in spacetime curvature. For observers at the edge of the universe to perceive themselves at its center instead, spacetime curvature is required. However, as the universe expands, spacetime curvature straightens out. Being consistent with general relativity would be a decrease in electrostatic energy along with gravity, as with a relative increase in the radius of the atom for the observer's clock to be slower, and for measured distance to be less. With both decreases, the Cosmological Principle could then agree with nullification effects of relativity theory. For instance, perceptual decrease in size of the universe by means of a relatively slower clock and relatively longer unit of measure could counter the effect of expansion for it to be perceived consistent with Einstein's original proposal of a static and finite universe. It only entails a modification of relativity theory in indicating size is relative as well.

Other modification of cosmology has been the inclusions of Dark Energy and Dark Matter. Dark Energy has been assumed due to the data of astronomical observation differing from the Big Bang prediction of the Tolman Brightness Test. Dark Matter has now been assumed because of a long-time discrepancy of data indicating galaxies rotate faster than they should according to either Newton's inverse-square-law of gravity or Einstein's theory of general relativity as spacetime curvature. The Tolman Brightness Test was a deciding factor in discarding Tired Light in favor of Big Bang. In the 1990s, however, astronomical data did not appear to be in agreement with the Big Bang predictions. Eric J. Lerner of the Lawrenceville Plasma Physics Inc., USA, published an article in 2009 titled Preliminary Results Challenge the Expanding Universe Model. A study of data favored a non-expanding-universe model instead. What resulted was Big Bang being modified to comply by assuming some unknown source of Dark Energy has increased the rate of expansion from that of the past.

Dark Matter has been assumed to explain why rotations of some galaxies are of speeds greater than their gravitational mass allows according to present theory. It constitutes a modification of theory in the sense Dark Matter is an unknown element invisible to electromagnetic radiation to thereby result in change of effect to gravitational action. There have also been proposed direct-modifications of both Newtonian Mechanics of gravity and general relativity theory.

Horace Babcock, as early as 1939, noted a discrepancy in the rotational speed of the Andromeda galaxy. In the 1970s and 1980s, a general study of galaxies under the guidance of Vera Rubin indicated rotational speeds from their galactic centers generally do not appear to decrease in accordance with Newton's inverse-squarelaw. Mordehai Milgrom proposed a modification of the law in 1983, referred to as MOND. In 2004, Jacob Bernstein offered a relativistic version of the modification according to space-time curvature. In 2008, John W. Moffat published his book Reinventing Gravity regarding modification of gravity (MOG) whereby the gravitational constant relatively increases farther away from the center of mass due to greater decrease in repulsive force away from the center of mass.

By present gravitational theory, the orbital speed of a planet around the sun decreases for greater radial distance. However, the surface of a large spherical mass has more gravitational force outward than inward because of gravitational influence in opposite directions cancelling each other out. Gravity is essentially a condition of homogeneity. Consider, then, the growth of a large spherical mass wherefrom the there is greater rotational speed at its surface than more towards its center. The perpendicular direction from its rotational center thus tends to become more of a disk than a sphere. Somewhat in between these two extremes of disk and sphere is a constant orbital speed of radial distance.

Further consider how cosmic mass is observed according to visible radiation. Of particular relevance is that eighty percent of the mass in the universe has been estimated to be hydrogen. Cold hydrogen is relevant in that its presence is extremely difficult to detect. The electron of the hydrogen atom, as assumed, can either rotate in the same direction or the opposite direction that the proton in the nucleus of the atom rotates. It is change of rotational direction from same to opposite direction that describes events of radiation, whereby low light-energy is detected, as typical of a Cosmic Microwave Background Radiation having no evidence of its direction

of origin. Undetected cold hydrogen could thus very well be the missing mass in the spiral galaxies, and it could also be part of the creation of new stars.

Justification for the consideration of this possibility in view of Tired Light is that it provides a long-range-vacuum-effect in the wake of emitted radiation for the explaining of a relatively weak force of gravity compared to atomic forces, whereby the average mass-energy density of the universe per nuclear density of the hydrogen atom equates to the ratio of gravitational force to electromagnetic force between the proton and electron of the hydrogen atom.

Tired Light

When Hubble and Humason formulated the Hubble Constant in 1929, Fritz Zwicky (1898-1974) proposed an explanation in terms of Tired Light. It assumes light loses energy by collisions with mass particles along its journey through space. For instance, electrons of cosmic plasma could more easily detach from atoms than do electrons of ordinary matter. Light would then decrease in energy by inelastic collision. However, the Zwicky proposal was dismissed for two reasons: 1. At the time, it was believed to have failed the Tolman Brightness Test of distinguishing between decreased energy effect in favor of the Doppler one of recessional speed; 2. No appreciable blurring of distant sources of light had been detected as expected of light interacting with intergalactic particles in space.

The blurring condition was acknowledged by Zwicky, but explanation as to why it does not occur has been offered, as by Einstein and other physicists. Einstein proposed, in relation to the wave-particle duality paradox, photons are of invisible waves with a minute particle of mass within them. Such speculation still needs to explain how an invisible wave can guide the particle for it to lose energy and still be invisible. Somewhat more plausible is the probability analysis of Richard Feynman. Accordingly, there is a slight probability of collision according to the average density of the universe. The slight probability of collision pertains to both the reflection of light and its redshift decrease in energy. There is thus a loss of light intensity along with the redshift. According to Feynman, the result is that the redshift distance is seen as longer because of the decrease in light intensity of longer distance.

As for Tired Light, losing energy during propagation, in contrast to Doppler Effect of Big Bang, is still somewhat speculative as to exactly how the energy is absorbed. If it is absorbed in proportion to light-energy, the value of the Hubble Constant could decrease with a decrease in light-energy along with an increase in distance. However, if the space-medium absorbs only a constant quantity of energy, then the Hubble Constant could truly be constant. There is also Feynman's probability condition to consider with slightly different interpretation. The decrease in intensity could differ from redshift in that intensity relates to surface area and distance squared. The Planck Constant per wavelength relates as momentum, but if the interaction of light with its medium is kinetic, a to accelerate whatever it collides with, it then relates to frequency and energy, being quadratic. The Hubble Constant could then be constant as well.

Tired Light and Gravity

A New Tired Light theory has been proposed by Lyndon Ashmore that is conditional to Quantum Mechanics and a Hubble Constant of an average value that has been determine by various astronomical observations over the years. His Hubble Constant H₁ is thus a mean value of various values of H. It relates in the manner

$$H_1 = \frac{hr_e}{m_e k} \approx 64 \frac{km}{(sec)(Mpc)} \tag{1}$$

 H_1 is Ashmore's value of the Hubble Constant, m_e is the electron mass, as determined according to experiment, and r_e represents a particular value of the electron radius, as consistent with theory. The constant k has a value of one cubic meter according to Ashmore, as an average density of mass in the universe as has been determined by astronomical observation.

Although the constant k being equal to exactly one cubic meter seems a coincidence, it is consistent with the theory of general relativity inasmuch as the density of the universe in ratio to the nuclear density of the hydrogen atom equals the ratio of electrostatic and gravitational forces of the electron and proton in the hydrogen atom.

An average density ρ_u of the universe in relation to H₁, as according to Einstein's formula, is

$$\rho_u = \frac{3H_1^2}{8\pi G} = \frac{3c^2}{8\pi R_u^2 G} \approx \frac{3(2GM_u)}{8\pi R_u^3 G} = \frac{M_u}{\frac{4}{3}\pi R_u^3}$$
(2)

Since the dimensional value of the Hubble Constant refers to a velocity per distance, H_1 in the equation converts to light speed c for a critical radius R_u in relation to the gravitational escape velocity squared as $c^2 = 2GM_u/R_u$.

The density of the universe in ratio to the density of an atomic nucleus of the hydrogen atom further approximates to the ratio of the gravitational and electrostatic forces between the proton and electron in the manner

$$\frac{M_u}{\frac{4}{3}\pi R_u^3} \div \frac{m_p}{\frac{4}{3}\pi r_n^3} = \frac{M_u r_n^3}{m_p R_u^3} \cong \frac{Gm_p m_e}{e^2} = \frac{Gm_p}{r_e c^2}$$
(3)

The average density of our finite universe in ratio to the nuclear density of the hydrogen atom is thus approximately the same ratio of gravitational and electrostatic forces between electron mass m_e and proton mass m_p .

By substituting equalities of equation (1), equation (3) becomes

$$\frac{3H_1^2}{8\pi G} \div \frac{3m_p}{4\pi r_n^3} \cong \frac{Gm_p}{r_e c^2} \tag{4}$$

$$H_1^2 \cong \frac{2G^2 m_p^2}{r_n^3 r_e c^2} \tag{5}$$

Equation (5) is to be linked with another value of H.

As noted, H_1 represents an average value. It is also representative of an energy exchange between light and plasma. The decreased energy of light is assumed by Ashmore to be the Cosmic Microwave Background Radiation.

The CMBR explanation is reasonable, but if the average Doppler shift is to the red end of the spectrum, the CMBR needs to somehow recycle in order for it not to continually increase without limit. It could, for instance, provide the material needed to create new stars while old ones burn out of existence in a manner consistent with Steady State Cosmology, or with the Heisenberg probability condition of quantum physics.

As for a recycling of the CMBR, perhaps matter reabsorbs it in various ways. It could, for instance, be absorbed by virtual particles conditional to the probability condition of quantum mechanics. As some stars evaporate, others form from the plasma and CMBR moving every which way.

The overall process might also simulate as a cause of gravity. If energy is given to intergalactic plasma, then the emission of light by greater density of mass could result in a vacuum effect in its wake of emission. The emitted light gradually interacts with plasma whereby the plasma acquires energy for a quick refill of mass that had been converted to radiation.

A cosmic-gravitational coincidence is evident of the Hubble Constant and the mass and size of the hydrogen atom, as within determined accuracy of the values of parameters used in physics, which here given in dimensions of grams, centimeters and seconds are:

Values of Physical Parameters

Gravitational Constant: G = $6.67428(67) \times 10^{-8} \text{ cm}^3/(\text{gm})(\text{sec})^2$ Proton mass: $m_p = 1.67262137(13) \times 10^{-24} \text{ gm}$ Electron mass: $m_e = 9.10938215(45) \times 10^{-28} \text{ gm}$ Light speed c: $2.99792458 \times 10^{10} \text{ cm/sec}$ Electron radius: $r_e = 2.8179402894 \times 10^{-13} \text{ cm}$ Neutron radius: $r_n = 2.881991 \times 10^{-12} \text{ cm}$ Fine Structure Constant: $\delta = 1/137.036 = e^2/\hbar c = v/c$ Planck Constant: $2\pi\hbar = h = 6.626069 \text{ (gm)}(\text{cm})^2/\text{sec}$ Electronic unit of charge e squared: $e^2 = 23.07 \times 10^{-20} \text{ (gm)}(\text{cm})/(\text{sec})^2$ The above values are partly a simplification of theory rather than fact. The real value of the electron radius, for instance, could be two thirds r_e in accordance with the Thomson formula of $2e^2/3r_{e'} = m_ec^2$ instead of $e^2/r_e = m_ec^2$. The simplified formula merely relates more directly to the Bohr radius r_a of the hydrogen atom, which is $(137.036)^2$ times greater than the electron radius r_e of the hydrogen atom, such that $r_a(m_e/m_p) = r_n$ and so on.

The electrostatic unit of charge e has historically been associated with electric emissivity and magnetic permissibility constants, but these constants are here assumed to be part of the Hubble Constant in relation to a relative density and gravitational influence of the observable universe. This does not mean the constants are not meaningful to more complete understanding of theory. They are merely implied, for simplicity, with regard to light speed, as they combine as a product equal to light speed squared in vacuum space.

The Gravitational Cosmic Coincidence

Zwicky proposed the redshift in the more distant starlight is related to gravity. The redshift is also considered here as radiation absorbed by the medium of space as gravitational effect. Space not only absorbs light energy; the energy is further absorbed by matter for it to emit the energy as electromagnetic radiation whereby rate of decrease is coincidental to a ratio of gravitational and electrostatic forces.

There is thus a recycling process. The emitted radiation is a long range and partly elusive process determined according to the probability condition of quantum physics, as typical of Earth's detection of only a minute portion of the neutrinos passing through Earth. Part of the radiation thus escapes detection except for its gravitational-vacuum effect. As radiation is gradually detected, it converts into space inertia. Space inertia, in turn, superimposes on inertial mass for the continuance of the recycling process.

Consider a critical radius of an observable universe as $R_u = c/H$. As c is the upper limit for its escape velocity, a distance r for v is such that v is of the same proportion to c that r is to R_u . Coincidentally, again, a velocity and distance per light speed in relation to the hydrogen atom is the same as its ratio of gravitational to electromagnetic forces, or potentials. However, how it relates to the structure of mass has different interpretation. Since a proton has about 1836.15 times the mass of an electron, since the hydrogen atom is one proton and one electron, and since the proton is within the nucleus of the atom having a radius of about 1836.15 times shorter than the radius of the atom, as the approximate distance of the electron from the nucleus, the calculation approximates the same for relating a Hubble Constant either to a nuclear mass and its radius or to the atom and its radius.

The simpler relation is with regard to the latter in the manner

$$\frac{H_2(2r_a)}{c} \cong \frac{Gm_a^2}{e^2} \cong 8.08 \times 10^{-37} \tag{6}$$
Dividing and multiplying each side of the equation by r_a , and e^2 obtains

$$\frac{2H_2e^2}{c} \cong \frac{Gm_a^2}{r_a} \tag{7}$$

By substituting R_u for c/H_2 , equation (7) becomes

$$\frac{2e^2}{R_u} \cong \frac{Gm_a^2}{r_a} \tag{8}$$

By substituting parameters $m_a v^2 r_a$ of e^2 for e^2 , equation (8) becomes

$$\frac{2m_a v^2 r_a}{R_u} \cong \frac{Gm_a^2}{r_a} \tag{9}$$

By dividing both sides of equation (9) by \mathbf{r}_a , it becomes

$$\frac{2m_a v^2}{R_u} \cong \frac{Gm_a^2}{r_a^2} \tag{10}$$

The centripetal force of two hydrogen atoms at opposite ends of the center of the observable universe thus equates with the gravitational force between two atoms separated a distance equal to the diameter of the atom.

The similar relation with regard to the proton mass m_p of the nuclear radius r_n of the hydrogen atom is less simple. Note: Contrary to gravity, the electron's speed is not according to nuclear mass it circles; it is according to the electron's own mass. Ratio of gravitational to electromagnetic potential is thus according to proton mass for gravitational potential and to electron mass for electromagnetic potential in the manner.

$$\frac{H_3(2r_n)}{c} \cong \frac{Gm_p}{r_n} \div \frac{e^2}{m_e r_n} = \frac{Gm_p m_e}{e^2} = \frac{Gm_p}{r_e c^2}$$
(11)

$$H_3 \cong \frac{Gm_p}{2r_e r_n c} \tag{12}$$

$$H_3^2 \cong \frac{G^2 m_p^2}{4r_e^2 r_n^2 c^2}$$
(13)

The ratio of Ashmore's value of H₁ squared to H₃ squared is

$$\frac{H_1^2}{H_3^2} = \frac{2G^2m_p^2}{r_n^3 r_e c^2} \div \frac{G^2m_p^2}{4r_e^2 r_n^2 c^2} = \frac{8r_e}{r_n}$$
(14)

The respective radii relate to internal and potential energies as

$$r_e = \frac{e^2}{m_e c^2} \tag{15}$$

$$r_n = \frac{e^2}{m_p \nu^2} \tag{16}$$

Hence

$$\frac{8r_e}{r_n} = \frac{8e^2}{m_e c^2} \div \frac{e^2}{m_p v^2} = \frac{8m_p v^2}{m_e c^2} \tag{17}$$

The ratio of the Hubble Constant values squared thus equate as 8 potential energies per internal energy of the electron.

The velocity v relates to the fine structure constant, as $v/c = e^2/\hbar c = 1/137.036$. The ratio squared to c indicates approximate relativistic effect:

$$1 + \frac{v^2}{c^2} \approx \frac{1}{1 - \frac{v^2}{c^2}}$$
(18)

It is significant with regard to the equivalence principle whereby free fall is according to a relatively homogeneous field of gravity. (The numerical value of v in this case is also the fine structure constant equal to the electrostatic unit e squared in ratio to hc.)

The factor 8 is according to the Schwartzschild Metric for a maximum escape speed c calculating as the square root of one half c in the manner

$$c\left[1 - \frac{2GM_u}{R_u c^2}\right] = c\left[1 - \frac{1}{2}\right] = \frac{1}{2}c'$$
(19)

Moreover, in ratio to the escape speed is the orbital speed c", such that

$$c = \frac{1}{2}c' = \frac{1}{2\sqrt{2}}c'' \tag{20}$$

$$c^2 = \frac{1}{4}c'^2 = \frac{1}{8}c''^2 \tag{21}$$

The factor 8 is thus explained as relativistic effect of gravitational potential of mass approaching one eighth light speed whereby light speed in the field becomes one half of c as an upper limit.

The application of the orbital speed is here explained as emerging with the escape speed as the gravitational field becomes homogeneous regarding the cosmic

scale as large. Because of spacetime curvature, light speed along with the curved path complies with the Cosmological Principle. It has been estimated from astronomical observations that a cosmic scale for a cosmic homogeneity is 250 million light years (55.2 times smaller than a perceived size of our universe.

With regard to the ratio of densities, interrelating equations (2), (4), (5), (13), (14), (17) and (21) with v" in relation to c" obtains

$$\frac{3H_3^2}{8\pi G} \div \frac{3m_p}{4\pi r_n^3} \cong \frac{Gm_p m_e}{e^2} \cdot \frac{m_e c^2}{8m_p v^2} = \frac{Gm_e^2 c^2}{8e^2 v^2} = \frac{Gm_e}{8r_e v^2} = \frac{Gm_e}{r_e v^{"2}}$$
(22)

The density of the observable universe in ratio to the nuclear density of the hydrogen atom thus approximates to the ratio of the electron's gravitational potential to its orbital-speed-squared around the nucleus of the atom.

Arbitrary Mass Charge and Gravity

Gravity and electromagnetism differ but equate in ratio to a particular mass unit as a common constant for both gravity and electromagnetism:

$$\frac{e^2}{G} = m^2 \cong 3.442 \times 10^{-12} \text{ grams squared}$$

However, it is further evident a mathematical relation of these effects, with regard to their proportionality, requires a particular mass unit as a coupling constant. In this regard, the mass unit can relate to a particular time unit in relation to the Hubble Constant. Consider one gram as the mass unit of the gravitational constant G, one second as its time unit, and one centimeter as its distance unit.

A gravitational force of one gram of mass at a radius of one centimeter per light speed relates in the manner

$$\frac{Gm_1}{r_1^2c} = \frac{6.674 \times 10^{-8}}{3 \times 10^{10} (second)} = 2.237 \times 10^{-18} \text{ per second}$$

Multiplying this value by a million parsecs (3.09×10^{19} kilometers) equates it to a Hubble Constant of 69.12 kilometers per second per million parsecs.

A three percent increase in the one=gram unit of mass renders a value equal to H₃ for it to be consistent with equation (11). Choosing such a unit is arbitrary, but it still does illustrate how a chosen unit can relate to theory. Further choice of units for distance and time, for instance, could equate the gravitational constant to the electrostatic unit squared $G/e^2 = m^2$. If m is a gram, then $e^2/G \approx 3.46 \times 10^{-12}$ grams squared. By decreasing the gram unit by the square root of that result, then G and e^2 numerically equate, but then the Hubble relation is sacrificed. For it to be

maintained, a change in time and distance units needs to be considered whereby the numerical Hubble value of per time also equates.

A difference in mass unit generally does not determine a difference in magnitudes of an outcome. The ratio of the proton mass m_p to the electron mass m_e remains the same for a mass unit of one half gram as it does for a gram. The ratio of Planck constant h to electrostatic unit e squared does not change, as both relate to the same mass unit whatever its numerical value: $\hbar = mvr$ and $e^2 = mv^2r$. Moreover, even though gravitational force increases per mass and electromagnetic force decreases per mass, gravitational energy and electromagnetic energy relate as massspeed-squared products whereby determined ratios of energies do not change by a change in a mass unit. For a greater mass unit, the gravitational constant must equally increase in order for a gravitational potential to maintain the same magnitude of v². If a mass unit m is doubled, then the numerical magnitude of m is halved. For Gm/r to still equal v², the numerical value of G needs to double as well. Although a doubling of the mass unit increases the electrostatic potential e^2/mr unless the numerical value of e² decreases instead of increases, the ratio of Gm²/e² remains the same due to a decrease in m^2 being nullified by both an increase in G and a decrease in e² because of the numerical value of the latter being inversely related to the numerical magnitude of the dimensionless quantity.

In relation to parameters mvr of the Planck Constant \hbar , consider v/c = 1/137 for a shorter distance by quicker light action, R the Schwartzschild radius for the limit of changes in speeds u equaling c, and Hr = u as change in speed and energy at distance r. For event horizon r, we have \hbar = mHRr = mvr. General relativity thus equates with the QED probability condition.

GRAVITY CAUSE EXPLAINED

Newton formulated gravitational force according to his inverse-square-law, but he was unable to explain the cause of gravity other than by an action at a distance principle. Einstein explained gravity as mass-energy following the path of spacetime curvature due to the presence of mass, but more entailed explanation of how the presence of mass causes spacetime curvature is still lacking. Here, gravity has been associated with the Hubble Constant insofar as a minute decrease in radiant energy with regard to its propagation in the medium of space allows for a long-range effect of a relatively weak force of gravity per mass in comparison to such other forces of nature as atomic and electromagnetic. However, although a vacuum effect is possible in the wake of emitted radiation, there is yet adequate explanation as to how a restoring force maintains equilibrium states of mass in manner of continuous action according to the Doppler principle of relative motion.

Explanation is here given in view of a virtual vacuum condition that is now an integral part of quantum physics. It assumes gravitational radiation is consistent with the tired light mechanism of space, whose main objection is a lack of explanation as to how space can decrease the energy of light and allow the visibility of the distant stars. How this visibility is possible is thus given viable explanation along with electric charge explanation as well.

Electromagnetism is part of the visibility explanation with regard to a righthand rule and a more causal explanation of interaction between virtual particles as proposed by Feynman with limited explanation. For instance, no causal explanation of how virtual particles cause attraction was deemed necessary according to Feynman. An explanation is here given as more causal with the inclusion of a concept of zero-point-energy (ZPA), which Plank later proposed as a modification of his original formulation of the quantum.

Gravitational radiation (gravitons) emitted for gravitational effect also are virtual particles, but the explanation includes more indebt analysis of the method of radiation superimposing to form observable mass as consistent with how mass relates to both relative motion and gravity.

Vacuum Effects

It has been argued primary substance would dissipate into empty space without any internal mechanism to form into particles if space were partially empty. Whether space is only partially filled or is a plenum, quantum theory now describes vacuum space as containing virtual energy particles according to the Heisenberg uncertainty principle. Such virtual particles as gluons are to explain observable effects that do not otherwise comply with predictions of theory. The gluon is confined as part of a proton or neutron such that it cannot be observed directly as an individual particle apart from a proton or neutron. It is verifiable only as an indirect effect according to mathematical analysis. It thus seemingly exists as a virtual particle. In general, the vacuum of space is now assumed to contain an assortment of virtual particles.

This quantum vacuum condition is not here contested; it is expanded to include non-quantum conditions of continuous change in motion as well. Matter at rest absorbs and emits discrete units of electromagnetic radiation as quanta, but quanta also vary according to the Doppler principle. Relative motion, gravity and electric charge all comply with the Doppler principle of continuous change in effect.

Electrostatic and gravitational effects are explained as vacuum effects occurring in the wake of emitted radiation. Even though effects are visible, gravitons are virtual particles. Although ordinary light is a visible part of the electromagnetic spectrum, as x-rays and radio waves are directly detectable, virtual particles can explain electromagnetic effects as well. Both gravity and electromagnetism thus associate with a 'virtual-vacuum-cause-and-effect of electromagnetic radiation.

The virtual explaining of electromagnetic effects is in view of Feynman diagrams, but continuous change in force is explained in accordance with a concept of zero-point-energy Planck proposed to modify his own quantum theory for it to comply with the classical theories of continuous change. His effort was continued with proposed casual explanations of the particle-wave paradox by De-Broglie, a hidden variable approach by David Bohm (1917-1992) and a stochastic interpretation of quantum probability conditions by Jean-Pierre Vigier (1920-2004).

Plank revolutionized physics in the year 1900 with the introduction of the quantum as a solution to an infinity paradox of blackbody radiation, but he did not accept some of its implications. He continued to pursue a more consistent solution with classical electromagnetism. He contrived a possible solution in 1911 that assumed quantum effects are the particular oscillation mode of the atom. However, his assumption contrasted with Bohr's atomic theory whereby quantum jumps of discreet energy occur with absorption of radiation as well as its emission. In effect, the continuous manner of change in the relative motion of mass only occurs by reflecting radiation instead of by either absorbing or emitting it. However, a particular aspect of Planck's new theory did receive recognition.

Plank added the term $(\frac{1}{2})hv$ to his original equation relating energy of radiation to absolute temperature. He referred to this term as a zero-point-energy of an oscillator, such that the average energy at absolute temperature zero is not itself zero. Walther Nerst (1864-1941), who had formulated the third law of thermodynamics, reinterpreted the term in consideration of the possible heat death due to the loss of radiation emitted out of the universe. He compared the half quantum frequency $(\frac{1}{2})hv$ to temperature k_BT, where k_B is the Boltzmann constant also used for statistical analyses of the classical theory of kinetics. Further consideration of Plank's additional term became evident in view of Heisenberg's uncertainty principle in that otherwise zero energy at absolute zero temperature contradicts the principle in referring to possible determination of exact energy for any particular time.

Any possible frequency of radiation suggests there is a possible infinite magnitude of ZPE. However, the uncertainty principle further suggests an infinite magnitude of energy is undetectable with regard to a particular time and location being uncertain. A possibility of this uncertainty is explainable as invisible effects of interaction between virtual energies, as similar to how thermodynamic entropy is explainable as no change occurring between two systems of the same temperature. For instance, gravity is essentially invisible except for its gravitational effect because it is able along with light waves in general to occupy the same space, whereas matter supposedly cannot. Such invisibility is typical of wave action. Waves superimpose to produce visible effect only if the medium of wave action changes in a way it can be seen. If action within a medium is counterbalanced, then a direct change occurring within the interaction need not be seen beyond it.

A connection between ZPE and continuous change is with regard to a particlewave paradox. The photoelectric effect revealed that electrons freed by radiation are according to frequency instead of the intensity of radiation. Einstein explained this result as particle effects of electromagnetic radiation. The particles were referred to as photons, as distinguished from particles of matter. However, further experimental evidence of interference supported a wave interpretation of light, and the photoelectric effect can be explained as according to light frequency instead of light intensity.

Frequency is also a wave property. The higher frequency light can free more electrons than does the same energy of less frequent and more intense light because less frequent light converts its energy as molecular motion of heat, as do microwave ovens using lower energy microwave radiation.

Einstein later offered an explanation of photons guided by waves. The waves would be directly invisible to us, but a particle guided by a packet of waves interfering within themselves could explain the particle-wave duality. With regard to the existence of the particle, De Broglie considered particle effects as resulting from overlapping waves in analogy to the beats of sound occurring from sound waves. Schrodinger then developed De Broglie's idea in a consistent manner of Plank's attempt to relate the quantum to classical electrodynamics and relativity theory. Moreover, in 1954, Bohm and Vigier mathematically developed a casual wave-particle duality explanation, but the stricter indeterminism interpretation of the Heisenberg uncertainty principle prevailed instead in only explaining effects in accordance with conditions of probability with no further need of causal explanation.

A primary distinction between such electromagnetic waves as light and mass is that the former superimposes whereas the latter cannot occupy the same space. To the extreme, light is invisible to other light. However, light contains momentum and converts to mass in increasing the speed of mass by either its absorption or reflection from mass. The action can be elastic or inelastic. Elastic collision is an opposite extreme of superposition. Reality is somewhere between these two extremes wherefrom various inelastic effects occur. Rather than light being reflected straight back from a metal, it can be partially converted to heat, or it can result in the emission of particles if the frequency is too fast for the metal to absorb it thermodynamically.

Virtual Spin and the Right-Hand Rule

In relation to the Feynman diagrams, there are particles responsible for attraction and repulsion. Virtual particles repel electrons from electrons and protons from protons, and they attract electrons to protons and protons to electrons. Various effects arise. For instance, two parallel wires, as Ampere discovered, contract if they both have electric currents flowing in the same direction and repel if the currents flow in opposite directions. The Feynman diagrams suggest an explanation according to virtual particles, which is here to be included, but it is itself an underlying explanation for a right-hand-rule explanation.

A right-hand-rule explanation of electrical attraction and repulsion of two electric currents is in connection with a bipolar property of magnetism. Magnets are bipolarized wherefrom like poles repel each other and opposite poles attract each other. If the magnet is divided, each part obtains opposite poles. Electromagnetic waves are the continuation of electromagnetic fields and magnetic fields resisting the other. In short, the flow of current and the electric and magnetic fields are all perpendicular to each other in manner of the directions in which the hand and thumb point and the fingers curl. The opposite poles of magnetic effect of currents flowing in the same direction thus align closer to each other to attract, but like poles align by the currents flowing in the opposite directions for them to repel each other.

Why is there a right-hand rule?

Likely explanation pertains to chiral symmetry by which atoms tend to align in the directions of currents. They thereby spin in the same directions if currents flow in the same direction, and they spin in opposite directions if the currents flow in opposite directions. Electrons are thus displaced by the currents in perpendicular directions according to the directions of flow. The displacement direction is the same for the flow of two currents in the same direction and opposite for two currents flowing in opposite directions. An electron displacement in the same direction results in electrons and protons being closer between the wires for attraction, whereas electrons displaced in opposite directions results in them being closer for repulsion.

Explanation of this contraction and repulsion according to the virtual field of energy is according to the law of momentum. The electrons flowing in the same direction tend to emit virtual particles with more total momenta in the same direction and less total momenta perpendicular to the stationary wires. The virtual particles in turn collide to emit secondary virtual particles in the perpendicular direction to those of the wires. Secondary collisions are of less energetic virtual particles if from the collisions of the virtual particles moving more in the same direction than more in the opposite directions of motion.

Regard this explanation as fundamental on a primary level from which attraction and repulsion become functional on subatomic and atomic levels. The righthand rule is thus according to a particular reality that has aligned according to the right-hand rule. There could also be an opposite alignment according to a left-hand rule, which would constitute antimatter. The worlds often interact whereby one is an anomaly of the other.

Distant Visibility

The reason here for explaining electromagnetism is with regard to the need of explaining the visibility of the distant stars according to a tired light theory. An analogy for the explanation is with regard to television. How is it possible that images are transmitted through wires for countless viewers to clearly see?

The explanation given here is with regard to bar magnets having the ability to divide into multiple magnets.

The video of television consists of the collection of images by cameras that are transmitted as electromagnetic signals that propagate through both space and wires (or more clearly through cables). Required is a transmission of signals to be consistent with how the human brain distinguishes the data it receives. In general, a visible image maintains in the human brain for only a tenth of a second for each pictorial change given by the light source. Ten different images per second are what the brain comprehends with regard to a continuous sequence of scenery. In practice, between 25 and 30 complete pictures per second are received by the brain, with each picture divided into about 200,000 elements, or pixels. About two million individual details are thus perceived in total by the brain per second.

For transmission of data to be consistent with perceptual ability of the brain the signals need to be within a frequency range as bandwidth. Entailed in the process are antennas to both transmit and receive, and transformers. The signals are somehow amplified and multiplied for numerous observers. Signals themselves are electromagnetic waves as parts of an electric current surrounded by an induced magnetic field. How all individual images can be maintained is explainable if we consider the magnetic field is divisible into a field of numerous parts similar to how a bar magnet divides into smaller bar magnets. Packets of bar magnets of particular images are thus amplified and duplicated for alternate routes.

Significantly, individual arrangements of bar magnets do not change as long as their medium of propagation is in balance, even though their energy for propagation either reduces or is amplified. As noted, Maxwell explained the process of electromagnetic propagation as conditional to space alone, as not a part of any physical medium. Starlight being electromagnetic waves of energy can thus exist as bar-magnet-packets that maintain individual images while losing energy as they interact with the virtual field of energy by which they advance as wave action through space. What remains to be determined is whether starlight itself is amplified for revealing more individual, detailed information in similar manner of the broadcast of television.

Explaining Light Mass Energy

Einstein explained gravity as following spacetime curvature due to the presence of mass. What is mass? Einstein's mass-energy equation indicates mass relatively at rest is energy per light speed squared. For mass in relative motion, mass would become infinite mass-energy at light speed, but infinite energy is assumed not available to increase mass to light speed.

Light is assumed to differ from mass because it can superpose for it to occupy the same space, whereas mass cannot. Mass also varies in speed, but light speed in vacuum of gravitational free space is assumed constant. Light speed relatively decreases in a massive medium and in a gravitational field.

It is also assumed light and mass exist as two different forms of energy that convert from one form to another. Further assume mass is a particular form of light energy due to particular spacetime conditions. Superposing of light waves is thus limited. For explaining this limitation consistent with the relative motion of mass, assume electromagnetic energy becomes standing waves of mass from which relative motion of the standing waves is caused by further interaction with electromagnetic waves. If the packet of standing waves reflects in one direction more frequent waves than waves reflected in the opposite direction, a packet then obtains relative motion in the opposite direction in order to balance out the frequency of reflection.

It is further assumed all frequencies of virtual photons are available in the virtual field of vacuum space, but that their reflection by standing waves is conditional to the internal mechanism of the standing waves. The packets of standing waves thus allow superposing of electromagnetic waves to pass through undetected as virtual waves. The observance of nature is relative to how mass interacts with the virtual field of vacuum space.

There is also internal action of the wave packets to consider, as for the equivalence of gravitational and inertial mass. Mass can itself be a means for converting energy from one form to another. As it is bombarded by waves of energy that it absorbs and converts, additional effects of nature as those of electromagnetism and gravity are created.

More in general it is assumed vacuum space is not empty, as it contains instead a virtual field of virtual energy whereby invisible particles moving at light speed are slowed to convert into another form of energy, mass, either in quantum form or continuous change as Doppler Effect.

As for non-detection of radiation by matter, in quantum physics there are the virtual forms of energy that are detectable according to a Born rule, a quantum wave function interpreted as a probability amplitude (measure of change) for detecting an atomic particle within some particular time and/or a particular location. A probability of the detection can be extremely slight. Billions of neutrinos, for instance, move through our bodies every second, while only a few neutrinos are detected by all of Earth. The neutrinos thus, as secondary effects, are virtually invisible for the most part.

Although wave equations are interpreted as quantum probabilities, the wave properties are still fundamental

Although quantum refers to discrete units of energy, the properties of waves are still fundamental. Waves can also have momentum, as for an iron ball directly striking of one end of a row of touching iron balls. An impulse action through the row of balls moves as a wave of momentum through the row for the ball at the other end to move forward unimpeded and maintain the momentum forward.

Light waves are consistent both with quantum and wave effect in that the Plank constant consisting of parameters $mv(2\pi r)$ are maintained by light speed being constant whereby a change in light momentum mc is nullified by an equal but opposite change in magnitude of r. Shorter waves of higher frequency are thus more energetic in relation to greater mass.

For explaining the difference between light and mass, suppose mass is an equilibrium state of light waves moving at light speed and crossing paths from every direction. It is partly maintained by not allowing other waves to pass through it. It allows some of the waves to pass through while it reflects others in maintaining its form. Its form is itself capable of moving through space at different speeds, as does mass. How this free motion is possible is according to its particular state of equilibrium. If in motion, then it reflects the less energetic virtual waves counter to the direction of relative motion in allowing more energetic ones to pass through, and it reflects more energetic virtual waves arriving in the direction of relative motion in allowing the less energetic ones to pass through. In effect, mass is wavelike motion through quasi-vacuum space as its medium of propagation, which is opposite to the effect of reflection of observable radiation encountering mass. There is thus a range of observable electromagnetic waves that mass reflects and absorbs while allowing virtual electromagnetic waves to pass through. A predictive test is with regard to how a change in the view of the universe occurs due to a change in either relative motion or a gravitational field.

Suppose this new wave form of mass-energy has additional properties, such as further converting energy of the spacetime medium to gravitational energy, as constantly in proportion to the amount of mass according to the principle of equivalence. Furthermore, consider the probability of detecting this massless gravitational energy is remote for it to provide a vacuum effect in the wake of emission. It converts into a new form of energy that is only detectable as gravitational effect. Conservation of momentum is maintained because the extremely slight probability of its detection renders it as a long-range effect whereby the minute decrease in the momentum of the graviton equals the minute flow of virtual particles flowing in the opposite direction because of the vacuum effect. Mass is thus converted to virtual energy that is recycled in manner of converting into various forms of energy, which can further act in maintaining certain particle forms and change other particles into other particles.

Gravity Mass and Electric Charge

Instead of describing gravitational effect as force, Einstein interpreted it as spacetime curvature: the warping of space in relation to the presence of mass. However, he provided no explanation for the presence of mass. Mass itself remains somewhat of an enigma.

Mass and light interrelate according to Einstein's mass-energy equation $E = mc^2$. How mass contains this tremendous amount of energy is in itself of significance. Consider a spherical container of energy in equilibrium with its environment. Outside pressure of light on the wall of the container is the same as the pressure from light inside the container. If its radius is doubled, then the light inside the container moves twice the distance to strike surface area four times as great. There should be eight times less inside pressure in ratio to four times more outside pressure. A greater atomic energy thus somehow exists within the nucleus that somehow needs to be contained by the outer part of the atom.

The weaker electron field of energy cannot itself maintain the stronger energy field of the nucleus. How both fields are contained must thus relate to different sources of energy, as from the virtual field.

A plausible explanation of this containment is with regard to the mean free path principle proposed by Clausius in support of the kinetic theory of gasses. More mass interferes with the free path of escape. If the interference is in proportion to mass radius, such that the nucleus of the hydrogen atom with one proton has a radius 1836.15 times smaller than the radius itself of the atom, and the proton is 1836.15 times more massive than electron mass surrounding it, then the nucleus has 1836.15 more mass interfering with the light speed for it to maintain equilibrium with its environment. The nuclear density in comparison to the atom beyond the nucleus is thus greater to the fourth power, which is consistent with the Stefan-Boltzmann fourth power law of thermodynamics in relating light intensity to temperature. Although the greater mass of the proton is explainable, not explained is its relation to gravity and electromagnetism. They differ in that gravity is a simple formula for attraction whereas electromagnetism is bipolar whereby positive charges and negative charges attract each other, and positive charge repels positive charge and negative charges repels negative charge. Gravity and electromagnetism also differ whereby increase in gravitational potential Gm/r increases along with an increase in m, but electromagnetic potential e²/mr decreases along with an increase in m.

For explanation of this difference, assume mass consists of packets of superposed standing waves. Wave packets reflect, absorb and emit radiation in creating the effects of nature. Within wave packets are quantum states of equilibrium contained by surrounding energy fields. By interacting with the energy field, they convert it into gravitons that gradually convert back to the field. However, the equilibrium state of the wave packet itself separates into states that reflect, absorb and emit different frequencies of radiation, as for the creation of positive and negative charges that tend to restore balance for the existence of neutral states.

Gravity and electric charge differ even though both partly result from the conversion of virtual particles into other virtual particles. A conversion of virtual particles into slightly detectable gravitons is that of gravity. To the contrary, the conversion of virtual particles into other virtual particles by an electron or proton results in fields of containment. An electron interacting with different types of virtual particles converts them into a positive field of force that not only surrounds the electron; a repulsive force occurs from its overlapping of multiple creations from other electrons. Positive force fields are likewise created by the proton. However, when the electron and proton fields overlap, the virtual particles convert back to their original forms. The nullification of repulsive charges thus further constitutes a recycling process and a vacuum state of attraction for opposite charge.

In effect, the proton inside the nucleus is in equilibrium with higher frequency radiation of the virtual field than is the electron, whereby the action between the proton and the electron is an indirect action of them as equilibrium states.

How, then, are neutrons created?

Neutrons are relatively unstable. A half-life of a free neutron is about fifteen minutes. Within the nucleus of the atom, neutrons are an intermediary for the interaction of virtual particles. The neutron can thus emit anti-neutrino to become a proton, such that the neutron existence is of various properties conditional to its environment.

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